HINTS & SOLUTIONS

Maths

1. $x + 3 > 0$ and $\quad x^2 + 3x + 2 \neq 0$

   $x > -3 \quad x \neq -1, -2$

   $\therefore$ domain $= (-3, \infty) - \{-1, -2\}$

2. $f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx,\ k \in R$

   $f(-x) = -kx = -f(x) \Rightarrow f(x)$ is odd function

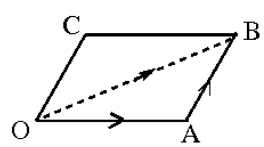
3. Given $S_n = \dfrac{4n^2 - 3n}{4}$

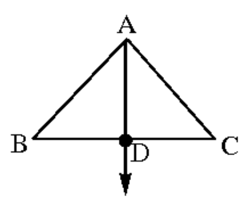
   $$t_n = S_n - S_{n-1} = \frac{4n^2 - 3n}{4} - \frac{4(n-1)^2 - 3(n-1)}{4}$$

   $$= \frac{4(2n-1) - 3}{4} = \frac{8n - 7}{4}$$

4. $\left.\begin{array}{l}\overline{OA} = i + 2J + 3K \\ \overline{AB} = 3i + 2J + K\end{array}\right\} \Rightarrow \overline{OB} = \overline{OA} + \overline{AB} = 4i + 4J + 4K$

   $\therefore$ Has direction unit vector $= \dfrac{\overline{OB}}{|\overline{OB}|} = \pm\dfrac{i + J + K}{\sqrt{3}}$



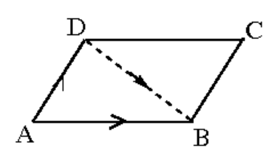
5. 

   $|\overline{AD}| = \left|\dfrac{\overline{AB} + \overline{AC}}{2}\right| = |4i - J + 4K| = \sqrt{33}$

6. $\overline{BD} = \overline{AD} - \overline{AB} = -(4\bar{a} + 5\bar{b})$

   Now $|\overline{BD}|^2 = 16|\bar{a}|^2 + 25|\bar{b}|^2 + 40|\bar{a}||\bar{b}|\cos(\bar{a}, \bar{b})$

   $= 128 + 225 + 243 = 593$



7. $\bar{a} = 3i - 5J \quad \Rightarrow |\bar{a}| = \sqrt{9 + 25} = \sqrt{34}$

   $\bar{b} = 6i + 3J \quad \Rightarrow |\bar{b}| = \sqrt{36 + 9} = \sqrt{45}$

   $$\bar{c} = \bar{a} \times \bar{b} = \begin{vmatrix} i & J & K \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix} = K\ (9 + 30) \qquad \Rightarrow |\bar{c}| = 39$$

   $\Rightarrow |\bar{a}| : |\bar{b}| : |\bar{c}| = \sqrt{34} : \sqrt{45} : 39$

8. $\Rightarrow [\bar{\alpha},\bar{\beta},\bar{\gamma}] = 0 \Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ $\Rightarrow a+b+c=0$

$\therefore \bar{v}.\bar{\alpha} = \bar{v}.\bar{\beta} = \bar{v}.\bar{\gamma} = a+b+c = 0$

9. Ratio of the roots $p : q \Rightarrow pqa^2 = ac(p+q)^2$

$$\Rightarrow \frac{(p+q)^2}{pq} = \frac{a}{c} \Rightarrow \sqrt{\left(\frac{q}{p}\right)} + \sqrt{\left(\frac{p}{q}\right)} = \sqrt{\frac{a}{c}}$$

10. roots real $\Rightarrow \Delta \geq 0$ $\Rightarrow a \leq 3$

$\alpha < 3$ $\Rightarrow a \pm \sqrt{3-a} < 3$ $\Rightarrow \pm\sqrt{3-a} < (3-a)$

$\Rightarrow \pm 1 < \sqrt{3-a}$

$\Rightarrow 3-a > 1$ $3-a > 0$

$\Rightarrow a < 2$ $a < 3$

11. $\left.\begin{array}{l} \sum \alpha = 0 \\ \sum \alpha\beta = 4 \\ \sum \alpha\beta\gamma = -1 \end{array}\right\}$ Now $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$

$$= \left(\frac{-1}{\gamma}\right) + \left(\frac{-1}{\alpha}\right) + \left(\frac{-1}{\beta}\right)$$

$$= -\frac{\sum \alpha\beta}{\alpha\beta\gamma} = 4$$

12. $|A^3| = 125$ $\Rightarrow |A| = 5$ $\Rightarrow \alpha^2 - 4 = 5$

$\Rightarrow \alpha = \pm 3$

13. $x^2 - 3x + 6 - 2x = 0$ $\Rightarrow x^2 - 5x + 6 = 0$ $\Rightarrow x = 2$ or 3

14. If A is a skew symetric odd order matrix $\Rightarrow |A| = 0$

$\therefore$ Inverse of A does not exists

15. Non-trivial solution $\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$, $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow (6-\lambda)\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1-\lambda & 2 \\ 1 & 3 & 1-\lambda \end{vmatrix} = 0$ $\Rightarrow \lambda = 6$

16. $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 1(2) + 1(0) + 1(2) = 4 \neq 0$

Rank = 3

17. $\frac{n(n-3)}{2} = 44$ $\Rightarrow n(n-3) = 88 = 11 \times 8$

$\Rightarrow n = 11$

18. ___ , ___ ,___ ,___ ,___ ,___ ,___ ,___ ,___ , $= \frac{4!}{2!2!} \times \frac{5!}{2! \times 3!}$

odd digits $\Rightarrow$ 3, 3, 5, 5 $= 6 \times 10$

even digits $\Rightarrow$ 2, 2, 8, 8, 8 $= 60$

19. $T_{r+1} = {}^{100}C_r \, 5^{\frac{100-r}{4}} \quad 4^{\frac{r}{5}}$

$\frac{100-r}{4}, \frac{r}{5}$ are integers if r = 0, 20, 40, 60, 80, 100

$\therefore$ number of rational terms = 6

20. $(1 + x)^6 [1 + (1 + x) + (1 + x)^2 + \ldots\ldots\ldots\ldots + (1 + x)^9]$

$= (1+x)^6 \left[\frac{(1+x)^{10} - 1}{1+x-1}\right]$

$= \frac{(1+x)^6}{x}[(1+x)^{10} - 1]$

$= \frac{(1+x)^{16}}{x} - \frac{(1+x)^6}{x}$

Req coet $= {}^{16}C_7 = {}^{16}C_9$

21. Let $S = \frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} \ldots\ldots\infty$

$\Rightarrow S = \frac{1.3}{1.2}.\left(\frac{1}{4}\right)^2 - \frac{1.3.5}{1.2.3}\left(\frac{1}{4}\right)^3 + \frac{1.3.5.7}{1.2.3.4}\left(\frac{1}{4}\right)^4 \ldots\ldots\infty$

$\frac{3}{4} + S = \left(1 + \frac{1}{2}\right)^{-1/2} = \left(\frac{3}{2}\right)^{-1/2} = \sqrt{\frac{2}{3}}$

$\Rightarrow S = \frac{\sqrt{2}}{3} - \frac{3}{4}$

22. $\frac{x^2 + 1 + 2x}{x(x^2+1)} = \frac{1}{x} + \frac{2}{x^2+1} = \frac{A}{x} + \frac{Bx + c}{x^2+1}$

$\Rightarrow A = 1, C = 2$ $\Rightarrow \cos^{-1}\left(\frac{A}{C}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

23. $\sum_{n=1}^{\infty} \frac{2^n}{n!} = \frac{1}{1!}.2 + \frac{1}{2!}(2)^2 + \frac{1}{3!}(2)^3 + \ldots\ldots\ldots + \frac{1}{n!}.2^n$

$= e^2 - 1$

24. A(adj A) = |A|.I $\Rightarrow$ |A| = 4

Now adj(adj A) = $(|A|)^{n-2}$ .A

$= 4^{3-2}$ .A

= 4A

25. $\cos\theta.\cos 2\theta.\cos 2^2\theta.........\cos 2^{n-1}\theta$

$$= \frac{\sin(2^n\theta)}{2^n.\sin\theta} = \frac{\sin\left(\frac{2^n\pi}{2^n+1}\right)}{2^4.\sin\left(\frac{\pi}{2^n+1}\right)} = \frac{\sin\left(\frac{\pi}{2^n+1}\right)}{2^n\sin\left(\frac{\pi}{2^n+1}\right)} = \frac{1}{2^n}$$

26. $\frac{1+\tan\theta}{1-\tan\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

$\Rightarrow 1 + \tan\theta - 3\tan^2\theta - 3\tan^3\theta = (9\tan\theta - 3\tan^3\theta)(1-\tan\theta)$

$\Rightarrow 1 + \tan\theta - 3\tan^2\theta - 3\tan^3\theta = 9\tan\theta - 3\tan^3\theta - 9\tan^2\theta - 3\tan^4\theta$

$\Rightarrow 3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0$

$\Rightarrow \sum \tan\alpha = 0$

27. $\tan\theta + \frac{-1+\tan\theta}{1+\tan\theta} = 2$ $\quad\Rightarrow \tan\theta - \tan^2\theta - 1 + \tan\theta = 2 + 2\tan\theta$

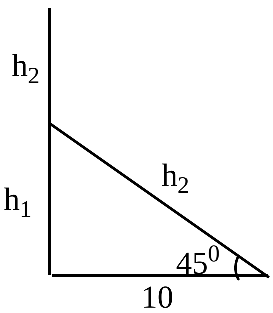
$\Rightarrow \tan^2\theta = 3 = (\sqrt{3})^2 \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

28. A + B + C = π $\quad\Rightarrow$ A + B = π – C

$\Rightarrow \frac{\tan A + \tan B}{1-\tan A\tan B} = -\tan C$ $\quad\Rightarrow \tan C = 1$ $\quad C = \frac{\pi}{4}$

29. $e^y = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$ $\quad\Rightarrow e^{-y} = \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

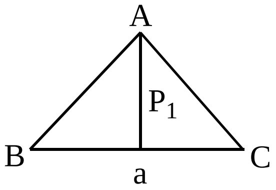
$e^y + e^{-y} = 2\left[\frac{1+\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]$ $\quad\Rightarrow \cosh\ y = \frac{1}{\cos x}$ $\quad\Rightarrow \cos x.\cosh y = 1$

30. 

$\Rightarrow h_1 = 10,\quad h_2^2 = 200 \quad\Rightarrow h_2 = \sqrt{2}\times 10$

$\therefore$ Req height = $h_1 + h_2 = 10(\sqrt{2}+1)$

31. 2R. tanA = 2R tanB = 2R tanc $\Rightarrow \Delta^{le}$ is equilateral

32. 

$\Delta = \frac{1}{2}a.p_1 \quad \Rightarrow \frac{1}{P_1} = \frac{a}{2\Delta} \quad$ Similarly $\frac{1}{P_2} = \frac{b}{2\Delta} . \frac{1}{P_3} = \frac{C}{2\Delta}$

But $\Rightarrow 2P_2 = P_1 + P_3 \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow$ H.P.

33. $x^2 + x + 1 = 0 \Rightarrow x = \omega$ or ω^2

Put $x = \omega \Rightarrow x + \frac{1}{x} = \omega + \omega^2 = -1$, $\Rightarrow x^2 + \frac{1}{x^2} = \omega^2 + \omega = -1$

$\Rightarrow x^3 + \frac{1}{x^3} = 1 + 1 = 2$

Reg = (1 + 1 + 4) + ... + (1 + 1 + 4)
= 6 × 9 = 54

34. $\alpha = \frac{2 \pm \sqrt{4-16}}{2} = 1 \pm i\sqrt{3} = 2\left(\frac{1}{2} \pm i\frac{\sqrt{3}}{2}\right)$

$\alpha = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = \beta = 2(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})$

$\Rightarrow \alpha^4 + \beta^4 = 2^4\left(2\cos\frac{n\pi}{3}\right) = 2^{n+1}\ \mathrm{Cos}\frac{n\pi}{3}$

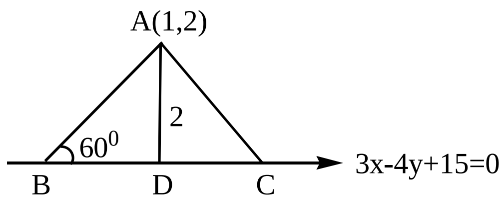
35. $\sin n\theta = {}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3}\theta \sin\theta + {}^nC_5 \cos^{n-5}\theta \sin^5\theta$..................

$\sin 6\theta = 6\cos^5\theta \sin\theta - 20\cos^3\theta \sin^3\theta + 6\cos\theta \sin^5\theta$

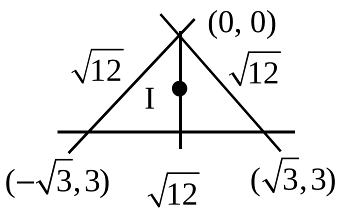
Now a + b + C = 6 – 20 + 6 =–8

36. $(y + 2)^2 = 4(x - 1)$ $y^2 = 4x$

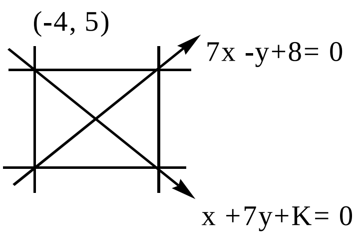
37. Length of AD = 2



$\therefore$ Length of the side = $\frac{2 \times 2}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

38. 

Inentre = centroid (G) = (0, 2)

39. 

$\Rightarrow$ K = – 31

40. $x - y + 1 = 0$ $\Rightarrow m = 1$ $\Rightarrow \theta = 45^0$

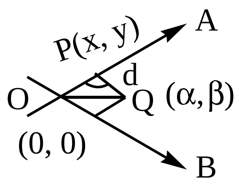
$P(x_1\ y_1) = (2, 3)$ Let the Req distance be r

$$\Rightarrow \left(2 + r.\frac{1}{\sqrt{2}}.\ 3 + r.\frac{1}{\sqrt{2}}\right)$$ Lie on $2x - 3y + 9 = 0$

$$\Rightarrow 4 + r\sqrt{2} - 9 - \frac{3r}{\sqrt{2} + 9} + 9 = 0$$

$$\Rightarrow 4 - \frac{r}{\sqrt{2}} = 0 \qquad \Rightarrow r = 4\sqrt{2}$$

41. Let $P(x_1\ y_1)$ be any point on the Req locus, $OA \perp PQ$



$$\Rightarrow \text{Area of } \Delta^{le} OPQ = \frac{1}{2}|\beta x - \alpha y| = \frac{1}{2} \times OP \times d$$

$$\Rightarrow (\beta x - \alpha y)^2 = (x^2 + y^2)d^2$$

42. $AB = \sqrt{1^2 + 4 + 36} = \sqrt{41}$

$BC = \sqrt{4 + 1 + 1} = \sqrt{6}$ $\Rightarrow AB^2 = BC^2 = AC^2$

$AC = \sqrt{1 + 9 + 25} = \sqrt{35}$ $\Rightarrow$ Req circumradius $= \dfrac{\sqrt{41}}{2}$

43. $\overline{AB}$ dr's $\Rightarrow (l_1\ m_1\ n_1) = (2, 2, 2)$

$\overline{CD}$ dr's $\Rightarrow (l_2\ m_2\ n_2) = (x - 7, 4, 1)$

$\overline{AB} \perp^{lr} \overline{CD} \Rightarrow l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow 2(x - 7) + 8 + 2 = 0$ $\Rightarrow x = 2$

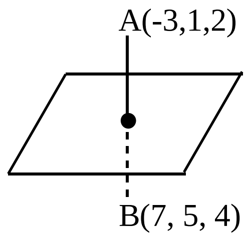
44. AB dr's $\Rightarrow (a, b, c) = (10, 4, 2)$

Mid point of AB = (2, 3, 3)

Req plane eq is $10(x - 2) + 4(y - 3) + 2(z - 3) = 0$

$\Rightarrow 10x + 4y + 2z = 20 + 12 + 6$

$\Rightarrow 5x + 24 + z - 19 = 0$



45. AB are conjugate point w.r.to the given circle

$$\Rightarrow AB^2 = 6^2 + 7^2 \Rightarrow AB = \sqrt{36 + 49} = \sqrt{85}$$

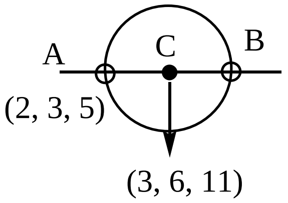
46. R.A. $\Rightarrow 6x + 14y + c + d = 0$ $(1, -4)$ lie on R.A.

$\Rightarrow 6 - 56 + c + d = 0$ $\Rightarrow c + d = 50$

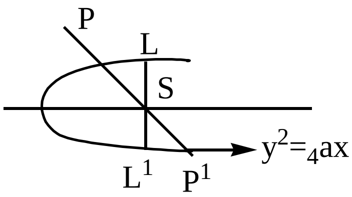
47. (1, 2) (4, 3) as diameter end points of a circle

$\Rightarrow (x - 1)(x - 4) + (y - 2)(y - 3) = 0$

$\Rightarrow x^2 + y^2 - 5x - 5y + 10 = 0$

48. 

B = 2C – A = (4, 9,–3)

49. 

We know by polar coordinates $\frac{1}{SP}+\frac{1}{SP^1}=\frac{2}{l}$

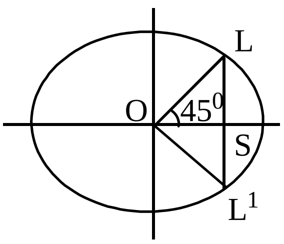
$\Rightarrow \frac{1}{SP}+\frac{1}{SP^1}=\frac{2}{SL}$ $\Rightarrow$ H.P

50.. Put X = x + a $\Rightarrow y^2 = 4aX$

Tangent becomes $\Rightarrow y = m(X - a) + C$

$= mX + C - am$

Now Req condition is $C - am = \frac{a}{m}$ $\Rightarrow C = a(m + \frac{1}{m})$

51. 

$\tan 45^0 = \frac{b^2/a}{ae}$ $\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0$ $\Rightarrow e = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5} \pm 1}{2}$

$\therefore$ e < 1 $\Rightarrow e = \frac{\sqrt{5}-1}{2}$

52. Req $\perp^{lr}$ distinces product $= \frac{\left|\frac{x_1}{a}-\frac{y_1}{b}\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} \cdot \frac{\left|\frac{x_1}{a}+\frac{y_1}{b}\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$

$= \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{a^2b^2}{a^2+b^2}$

53. $\frac{x^2}{r-2}+\frac{y^2}{5-r}=1$ represents an ellepse

$\Rightarrow r - 2 > 0, \quad 5 - r > 0$

$\Rightarrow 2 < r < 5$

54. $r = a\cos(\theta-\alpha)$ $\Rightarrow r^2 = ar[\cos\theta\cos\alpha + \sin\theta\ \sin\alpha]$

$\Rightarrow x^2 + y^2 - a\cos\alpha\ x - a\sin\alpha\ y = 0$

$r = b\sin(\theta-\alpha)$ $\Rightarrow r^2 = br[\sin\theta\cos\alpha - \cos\theta\ \sin\alpha]$

$\Rightarrow x^2 + y^2 - b\sin\alpha\ x - b\cos\alpha\ y = 0$

Now $2g_1g_2 + 2f_1f_2 = \frac{ab}{2}\cos\alpha\sin\alpha - \frac{ab}{2}\cos\alpha\sin\alpha = 0$

55. $\left.\begin{aligned} m &= \underset{\theta\to 0}{Lt}\frac{2\tan\theta}{\theta} = \underset{\theta\to 0}{Lt}\frac{\tan 2\theta}{\theta} = 2 \\ m &= \underset{\theta\to 0}{Lt}\frac{2\tan\theta}{\theta} = \underset{\theta\to 0}{Lt}\frac{\tan 2\theta}{\theta} = 2 \end{aligned}\right\}$ Req Quadratic equation is $\Rightarrow x^2 - (l+m)\,x + lm = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

56. f(x) discontinuous $\Rightarrow \log|x| \neq 0$ or undefine

$\Rightarrow |x| \neq 1$ or $x \neq 0$ $\Rightarrow x = \pm 1.0$

57. I) $f(x) = \frac{ax + a^2}{\sqrt{ax}}$ $\Rightarrow \sqrt{a}.\sqrt{x}.f(x) = ax + a^2$

$\Rightarrow \sqrt{a}.\frac{1}{2\sqrt{x}}.f(x) + \sqrt{a}.\sqrt{x}.f^1(x) = a$

at x = a $\Rightarrow \frac{1}{2}.f(a) + a\,f^1(a) = a$

$\Rightarrow a + a.\ f^1(a) = a$

$\Rightarrow f^1(a) = 0$

II) $1 - f(x) = (1 + x)^{-1}$ $\Rightarrow -f^1(x) = \frac{-1}{(1+x)^2}$ $\Rightarrow f^1(x) = \frac{1}{(1+x)^2}$

58. $y^2 = \sin x + y$ $\Rightarrow 2y.\frac{dy}{dx} - \frac{dy}{dx} = \cos x$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

59. $f(x) = \begin{cases} 1 + \sin x & \text{if} \quad x \geq 0 \\ 1 - \sin x & \text{if} \quad x < 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} \cos x & \text{if} \quad x \geq 0 \\ -\cos x & \text{if} \quad x < 0 \end{cases}$

Also $\underset{x\to 0^-}{Lt} f(x) = \underset{x\to 0^+}{Lt} f(x) = f(0)$

But $f'(0^-) \neq f'(0^+)$

60. Sin u$=\dfrac{x^2-y^2}{x^2+y^2}$ $\Rightarrow$ sin u is a homogenious function of degree n = 0

By Eular's formula $n.\dfrac{d}{dx}(\sin u)+4.\dfrac{d}{dy}(\sin u)=0.\sin u$

$x.U_x+y.U_y=0$

61. δr = 0.01 sec. r = 12 cm

Area A = πr^2 $\Rightarrow$ $\delta A=2\pi r\ \delta r=24\pi\times 0.01$

$=0.24\ \pi$ sq cmn/sec

62. δa = 0.05 cm a $\rightarrow$ side

Equilateral Δ^{le} Area $\Rightarrow A=\dfrac{\sqrt{3}}{4}a^2$

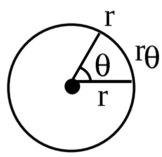
$\Rightarrow \log A=\log\sqrt{3}/4+2\log a$ $\Rightarrow \dfrac{dA}{A}=2.\dfrac{da}{a}$

$\Rightarrow \dfrac{dA}{A}\times 100=2.\dfrac{da}{a}\times 100=\dfrac{10}{a}$

63. $y^2=4ax$ $\Rightarrow 2y\dfrac{dy}{dx}=4a$ $\Rightarrow m=\dfrac{2a}{y_1}$ Let $P(x_1, y_1)$

$(y_1.m).\ y_1.\left(\dfrac{y_1}{m}\right)$ are in G.P.

64. $2r+r\theta$ = constant



$\Rightarrow r=\dfrac{K}{\theta+2}$ $A=\dfrac{1}{2}r^2\theta$ $\Rightarrow A=\dfrac{1}{2}.\dfrac{K^2\theta}{(\theta+2)^2}$

$\Rightarrow \dfrac{dA}{d\theta}=0$ $\Rightarrow \dfrac{K^2}{2}\left(\dfrac{(\theta+2)^2-\theta 2(\theta+2)}{(\theta+2)^4}\right)=0$

$\Rightarrow \theta=2$

65. $y=x^{n-1}.\log x$

$y_1=x^{n-1}.\dfrac{1}{x}+(n-1).x^{n-2}.\log x$

$xy_1=x^{n-1}+(n-1)y$

Diff. (n–1) times

$x.y_n+\cancel{(n-1).y_{n-1}}=(n-1)!+\cancel{(n-1)y_{n-1}}$

$y_n=\dfrac{(n-1)!}{x}$

66. $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx = \int \frac{(1-\sin^2 x)}{\sin x(1+\sin x)} \cos x dx$

$$= \int \frac{(1-\sin x)}{\sin x} \cos x dx$$

$$= \log|\sin x| - \sin x + c$$

67. Put $\text{Tan}^{-1}x = t \quad \Rightarrow \frac{dx}{1+x^2} = dt$

R.I. = $\int e^t(\text{Tan}t + \text{Sec}^2 t)dt$

$$= e^t \text{Tan } t + C$$

$$= xe^{\text{Tan}^{-1}x} + C$$

68. Period of x – [x] is 1

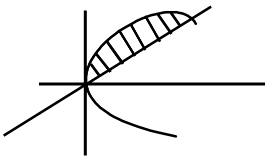
$$\int_0^{100} (x-[x])dx = 100.\int_0^1 (x-[x])dx$$

$$= 100.\int_0^1 x.dx$$

$$= 100\left(\frac{1}{2} - 0\right) = 50$$

69. $\underset{x \to 0}{\text{Lt}} \frac{\int_0^x \sin^3 t\, dt}{x^4} = \frac{1}{4}.\underset{x \to 0}{\text{Lt}} \frac{\sin^3 x}{x^3} = \frac{1}{4} = 0.25$

70. f(x) = $2x^3$– sinx $\quad \Rightarrow$ f(x) is odd function $\Rightarrow \int_{-a}^{a} f(x) = 0$

71.  Req Area = $\int_0^1 (\sqrt{x} - x)dx = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

72. $xy(ydx + xdy) + x^2y^2 (2ydx - xdy) = 0$

dividing with x^3y^3

$$\frac{d(xy)}{(xy)^2} + 2.\frac{1}{x}dx - \frac{1}{y}dy = 0$$

integrating $\frac{-1}{xy} + 2\log|x| - \log|y| = C$

73. $$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{(1+x^2)^2}$$

I.F =. $e^{\int \frac{2x}{1+x^2}dx}=1+x^2$

Solution $y(1+x^2) = \int \frac{1}{(1+x^2)^2}(1+x^2)dx$

$y(1+x^2) = \int \frac{1}{1+x^2}dx$

$y(1+x^2) = \tan^{-1} x + C$

at $x = 1$, $y = 0 \Rightarrow 0=\pi/4+C$ $\qquad \Rightarrow C=-\pi/4$

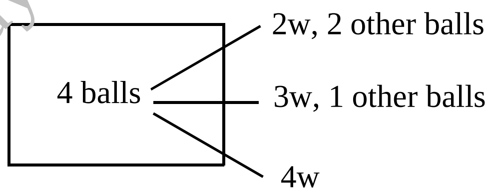
$y(1+x^2) = \tan^{-1}x - \pi/4$

74. R.P. $=\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right).\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+.......\infty$

$=\frac{5}{36}\left\{1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+...........\infty\right\}$

$=\frac{5}{36}.\frac{1}{1-\frac{25}{36}}=\frac{5}{11}$

75. R.P. $=\dfrac{\frac{1}{3}.{}^4C_2/{}^4C_2}{\frac{1}{3}.\frac{{}^2C_2}{{}^4C_2}+\frac{1}{3}.\frac{{}^3C_2}{{}^4C_2}+\frac{1}{3}\frac{{}^4C_2}{{}^4C_2}}$



= 3/5

76. $$\sum_{k=0}^{\infty} P(x=k)=1$$

$1.a+\frac{2}{3}.a+\frac{3}{3^2}a+\frac{4}{3^3}a+.......\infty=1$

$a.\left\{1+2.\frac{1}{3}+3.\frac{1}{3^2}+4.\frac{1}{3^3+}......\infty\right\}=1$

$a.\left\{\frac{1}{1-1/3}+\frac{1.1/3}{4/9}\right\}=1$

$a.\left\{3/2+\frac{3}{4}\right\}=1$

$a=\frac{4}{9}$

77. $p = \frac{1}{2}, q = \frac{1}{2}$ $\quad$ n = 4

$$P(x=4) = {}^4C_4\left(\frac{1}{2}\right)^4 = \frac{1}{2^4} = \frac{1}{16}$$

No. of families = $10,000 \times \frac{1}{16} = 625$

78. $h = \frac{1-0}{2} = \frac{1}{2}$ $\quad$ 0, 1/2, 1

$$\int_0^1 \frac{1}{1+x}dx = \frac{1}{4}\left\{\left(1+\frac{1}{2}\right) + 2.\left(\frac{2}{3}\right)\right\} = \frac{17}{24}$$

79. G.E. $\underset{n\to\infty}{\text{Lt}} \frac{1}{n^6} \cdot \sum_{r=1}^{n} (n+r)^5$

$$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\sum_{r=1}^{n}\left(1+\left(\frac{r}{n}\right)\right)^5 = \int_0^1 (1+x)^5 dx = \left\{\frac{(1+x)^6}{6}\right\}_0^1 = \frac{2^6-1}{6} = \frac{21}{2}$$

80. $x^2 + bx + 3 = 0$ $\quad$ $\Delta \geq 0$

$b^2 - 12 \geq 0$

b = 4, 5, 6

$\therefore$ R.P. = $\frac{3}{6} = 1/2$

PHYSICS

81. $\overline{\overline{A}+\overline{B}} = \overline{\overline{A}}.\overline{\overline{B}} = A.B$

82. for a given charge $V \propto \frac{1}{R}$

83. Conceptual

84. When α particle is emitted $Z\downarrow$ by 2 & $A\downarrow$ by 4

When β particle is emitted $Z\downarrow$ by 2 & A is unchanged.

When γ particle is emitted Z & A are unchanged.

85. According to L.C.L.M

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 + 0 = 0 + m_2v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{u_1} ----(1)$$

Given $k\in_f = \frac{1}{2}k\in_i$

$$\frac{1}{2}m_2v_2^2=\frac{1}{2}.\frac{1}{2}m_1u_1^2 \Rightarrow \frac{v_2^2}{u_1^2}=\frac{1}{2}\frac{m_1}{m_2}----(2)$$

from (1) & (2) $\frac{v_2}{u_1}=\frac{1}{2}=e$

86. $W=Fs=mas$

$$=ma\left(\frac{1}{2}at^2\right)=\frac{1}{2}ma^2t^2 -------(1)$$

Given $v=k\sqrt{s}$

$$\therefore a=\frac{dv}{dt}=k\frac{1}{2}s^{-\frac{1}{2}}\frac{ds}{dt}=\frac{k}{2}s^{-\frac{1}{2}}\left(k\sqrt{s}\right)=\frac{k^2}{2}-----(2)$$

From (1) & (2)

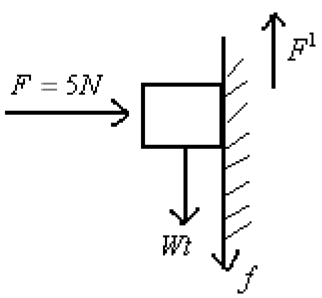
$$W=\frac{1}{8}mk^4t^2$$

87. $x_{cm}=\frac{m_1x_2+m_2x_2+.....+m_nx_n}{m_1+m_2+............+m_n}$

$$=\frac{m\ell+2m(2\ell)+.........+nm(n\ell)}{m+2m+..........+nm}$$

$$=\frac{\ell\left[1^2+2^2+............+n^2\right]}{1+2+..........+n}$$

$$=\ell\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\frac{(2n+1)\ell}{3}$$

88. 

here N = F

To move the duster upwards F^1 = wt + f

$$F'=Wt+\mu N=Wt+\mu F$$

$$F'=2+0.4\times5=4N$$

89. $2T_n = T_i + T_c \Rightarrow T_i = 2\times 270 - 10 = 530^0C$

90. PE at the top = KE at the bottom

$$Mgh = \frac{1}{2}MV^2$$

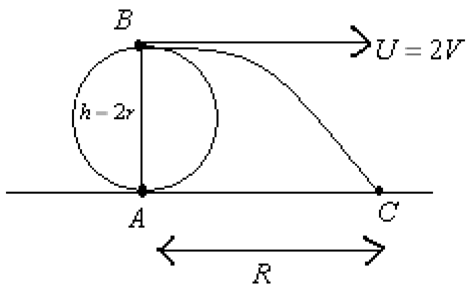
$$Mgh = \frac{1}{2}M\left(\sqrt{5gr}\right)^2$$

$$h = \frac{5r}{2}$$

91. AC = Range = ut

$$AC = 2V\sqrt{\frac{2h}{g}}$$

$$= 2V\sqrt{\frac{2(2r)}{g}} = 4V\sqrt{\frac{r}{g}}$$



92. $W = PE_f - PE_i$

$$= -\frac{GMm}{R+h} - \left[-\frac{GMm}{R}\right] = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+2R}\right] = \frac{2}{3}\frac{GMm}{R}$$

93. $$\frac{n}{n-1} = \sqrt{\frac{\ell_L}{\ell_S}}$$

$$\frac{n}{n-1} = \sqrt{\frac{121}{100}} = \frac{11}{10}$$

i.e., after the shortest pendulum completes 11 oscillations
(or)
After the longer pendulum completes 10 oscillations

94. $$\frac{KE_1}{KE_2} = \frac{h\vartheta_1 - h\vartheta_0}{h\vartheta_2 - h\vartheta_0} = \frac{2\vartheta_0 - \vartheta_0}{3\vartheta_0 - \vartheta_0} = \frac{1}{2}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{kE_1}{kE_2}} = \frac{1}{\sqrt{2}}$$

95. $$\ell = \frac{h}{\cos\alpha} = \frac{10}{\cos 60^0} = 20mm$$

96. Amount of fluid flowing per sec = $A\vartheta = \pi r^2 \vartheta$

$$\therefore r_1^2\vartheta_1 : r_2^2\vartheta_2 : r_3^2\vartheta_3 : r_4^2\vartheta_4 = 4:98:45:15$$

Ans: a,d,c,b

97. $V_L\gamma_L\Delta t = V_C\gamma_C\Delta t \Rightarrow V_L\gamma_L = V_C\gamma_C$

$$V_L \times 1.8\times10^{-4} = 2000\times27\times10^{-6}$$

$$V_L = 300\ cc$$

98. $P_1V_1 = P_2V_2$

$\Rightarrow (H+h)V_1 = HV_2$

$$(76+h)V_1 = 76\times\frac{3}{2}V_1$$

h = 38 cm

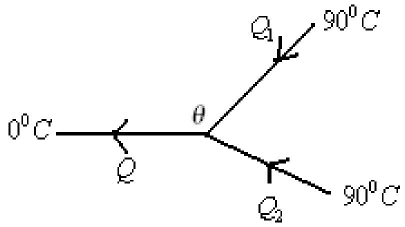
99. $dQ = du + dW$

$dQ = du + P\Delta V$ (work is done on the system $\therefore dW = -ve$)

$100 = du - 50\times(10-4)$

$du = 400J$ Increases

100.



$Q = Q_1 + Q_2$

$$\frac{KA(\theta-0)}{L} = \frac{KA(90-\theta)}{L} + \frac{KA(90-\theta)}{L}$$

$\theta = 180 - 2\theta$

$\theta = 60^0c$

101. $T_n = a + (N-1)d$

$2a = a + (64-1)4 \Rightarrow a = 252Hz$

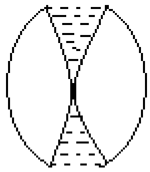
$T_{15} = a + (16-1)d = 252 + 15\times4 = 312Hz$

102. $n' = n\left[\frac{v-(-v_0)}{v-v_s}\right]$

$\frac{9}{8}n = n\left[\frac{v+x}{v-x}\right]$ $(v_0 = v_s = x)$

$\frac{9}{8} = \frac{340+x}{340-x} \Rightarrow x = 20m/s$

103. A : Image and scale magnified twice
B : Spherical abberation can be reduced by sharing deiviation at different surfaces.

104. 

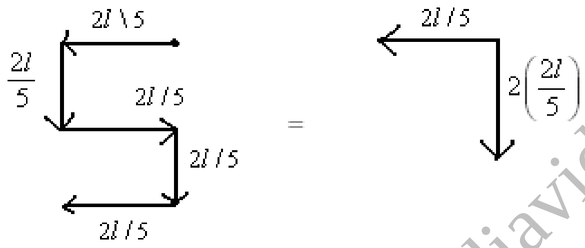
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$$

$$=(\mu_g-1)\frac{2}{R}\ \ -(\mu_w-1)\frac{2}{R}+(\mu_g-1)\frac{2}{R}$$

$$=0.5\times\frac{2}{20}-\frac{1}{3}\times\frac{2}{20}+0.5\times\frac{2}{20}$$

$$\frac{1}{F}=\frac{1}{20}-\frac{1}{30}+\frac{1}{20}=\frac{1}{15}\Rightarrow F=15cm,\ convex$$

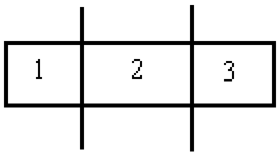
105. Dichroism = Double Refractin + selective absorption

106. 

$$R=\frac{2\ell}{5}\sqrt{1+4}=\frac{2\ell}{\sqrt{5}}$$

$$\therefore M'=\frac{m2\ell}{\sqrt{5}}=\frac{M}{\sqrt{5}}$$

107. $M'=\dfrac{M}{n}=\dfrac{M}{3}$ ------------ (1)



$$I'=\frac{m'\ell'^2}{12}=\frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2$$

$I'=\dfrac{I}{27}$ ------------- (2)

$$T\propto\sqrt{\frac{I}{M}}\Rightarrow\frac{T_1}{T_2}=\sqrt{\frac{I_1}{I_2}\frac{M_2}{M_1}}=\sqrt{\frac{I}{3I'}\frac{3M'}{M}}\ =\sqrt{\frac{I}{3\frac{I}{27}}\frac{3M}{3M}}$$

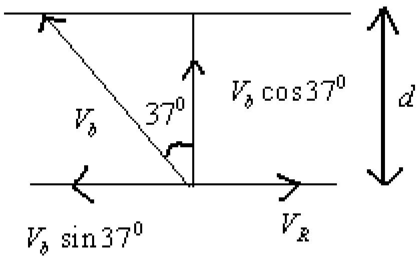
$$\frac{2}{T_2}=3\Rightarrow T_2=\frac{2}{3}\sec$$

108. $2\mu F \,||\, to 2\mu F$

$= 4\mu F$ and is serious to $12\mu F$

$= \dfrac{4 \times 12}{16} = 3\mu F$ and is parallel to $2\mu F$

$\therefore C_{AB} = 3 + 2 = 5\mu F$

109. 

drift (x) = $\left(V_R - V_b \sin 37^0\right) t$

$$= \left(V_R - V_b \sin 37^0\right) \frac{d}{V_b \cos 37^0}$$

$$= \left(3.5 - 5 \times \frac{3}{5}\right) \frac{60}{5 \times \frac{4}{5}}$$

$$= 0.5 \times 15 = 7.5m$$

110. Apply Kirchoff's first law at every junction

111. $\dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{\ell_1}{\ell_2} \Rightarrow \dfrac{E_1}{E_2} = \dfrac{\ell_1 + \ell_2}{\ell_1 - \ell_2} = \dfrac{800 + 600}{800 - 600} = \dfrac{7}{1}$

112. $F_{up} = 2F_{down}$

$$Mg\left[\sin\theta + \mu\cos\theta\right] = 2Mg\left[\sin\theta - \mu\cos\theta\right]$$

$$3\mu\cos\theta = \sin\theta$$

$$\tan\theta = 3\mu$$

$$\tan\theta = 3 \times \frac{1}{\sqrt{3}} \Rightarrow \theta = 60^0$$

113. $B_1 = \dfrac{\mu_0 i}{2(2R)} \qquad B_2 = \dfrac{\mu_0 i (2R)^2}{2\left((6R)^2 + (2R)^2\right)}$

$$\frac{B_1}{B_2} = \frac{\left(40R^2\right)^{3/2}}{(2R)^3} = \frac{80\sqrt{10}}{8} = 10\sqrt{10}$$

114. $S = \dfrac{G}{n-1}$

$n = \dfrac{i}{i_G} = \dfrac{20MA}{2MA} = 10 \quad \therefore S = \dfrac{180}{9} 20\Omega$

115. $e = B_v \ell V = 5\times 10^{-5} \times 20 \times \left(360 \times \dfrac{5}{18}\right) = 0.1V$

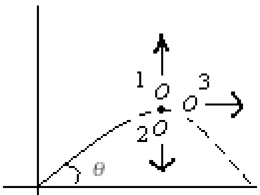
116. $X_L = WL = 3L \;\&\; R = 4L \qquad \therefore \mathrm{Z} = \sqrt{\mathrm{R}^2 X_L Y} = 5r$

$i_0 = \dfrac{E_0}{Z} = \dfrac{4}{5} = 0.8A$

117. $e = \dfrac{F\ell}{AY}\left[\begin{array}{l} m = Vd = A\ell d \\ \therefore A = \dfrac{m}{\ell d} \end{array}\right]$

$e = \dfrac{F\ell}{\left(\dfrac{m}{\ell d}\right)Y} = \dfrac{F\ell^2}{mdY}\left(F, Y \,\&\, d \text{ are same}\right)$

$\therefore e \propto \dfrac{\ell^2}{m} \Rightarrow \dfrac{e_1}{e} = \left(\dfrac{\ell_1}{\ell_2}\right)^2\left(\dfrac{m_2}{m_1}\right) = \dfrac{81}{100} \times \dfrac{4}{3} = \dfrac{27}{25}$

118. 

According to L.C.L.M

$mu\cos\theta = m_1 v_1 + m_2(-v_2) + m_3 v_3$

$m \times 200 \times \dfrac{1}{2} = \dfrac{m}{3} \times 100 - \dfrac{m}{3} \times 100 + \dfrac{m}{3} v_3$

V_3 = 300 m/s in horizontal direction

119. $P = \dfrac{F}{A} = \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$L = \dfrac{Q}{m} = \dfrac{ML^2T^{-2}}{M} = L^2T^{-2}$

Velocity gradient = $\dfrac{dv}{dx} = \dfrac{LT^{-1}}{L} = T^{-1}$

$\phi = BA = \dfrac{F}{m}A = \dfrac{MLT^{-2}L^2}{IL} = ML^2T^{-2}I^{-1}$

120. Given equation is of the form $y = Ax - Bx^2$

Range = $\frac{A}{B} = \frac{3}{\left(\frac{1}{8}\right)} = 24m$

$$H = \frac{A^2}{4B} = \frac{9}{4\times\frac{1}{8}} = 18m$$

CHEMISTRY

121. 1s can absorb energy, but does not emit

122. $\lambda \propto \frac{1}{m}$. Electron has least mass

123. Configuration of Cr is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5\ 4s^1$

124. O_2^{-2} has no unpaired electrons.

125. Na has larger size and forms cation easily

126. 99% Completion of first order reaction takes 6.7 half lives
99.9% completion requires 10 half lives.

127. $\frac{2r}{r} = \sqrt{\frac{M_B}{M_A}} \Rightarrow \frac{M_A}{M_B} = \frac{1}{4}$

128. Hydrogen bonding between molecules.

129. Oxidation state of N is +1 in N_2O, +2 in NO, +3 in N_2O_3 and +5 in N_2O_5

130. Change in oxidation state of Mn is 5.

131. $\text{Hardness} = \frac{\text{weightt of salt}}{\text{GMW}} \times \frac{10^8}{H_2O\text{ sample in ml}} = \frac{1.36}{136} \times \frac{10^8}{10^4} = 10^2$ ppm

132. PbO_2 is dioxide. Oxidation state of oxygen is -2

133. Br_2 cannot displace chloride.

134. Smallar the size of ion, more is the hydration ability.

135. Si forms four single bonds with sp^3 orbitals formed in excited state.

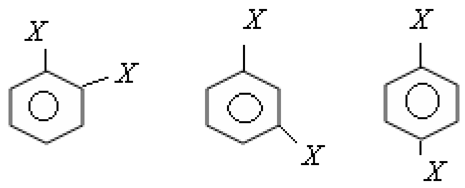
136. Liquid He is cryogenic. Beacon lights of Ne for vison through fog. Kr in helmet caps and Rn in cancer treatment ointments.

137. $NO + O_3 \rightarrow NO_2 + O_2$

138. CH_3 has 3 bond pairs and 1 lone pair

139. $x = C_2H_4$ and $y = C_2H_5OH$

$$C_2H_5OH \xrightarrow[350^0 c]{Al_2O_3} C_2H_4$$

140. 

141. $\Delta T_b = k_b \times \frac{w_1}{m_1} . \frac{1000}{w_2}$

$= 0.52 \times \frac{6}{60} \times \frac{1000}{90} = 0.577$

142. Only Po has simple cubic structure

143. Oxidation occurs at anode

$H_2 + 2OH^- \Rightarrow 2H_2O + 2e^-$

144. $k_p = P_{NH_3} . P_{H_2S}$ $\qquad$ $k_c = [NH_3][H_2S]$

units = atm. atm = atm^2 $\qquad$ units = $\frac{m}{l}\frac{m}{l}\left(\frac{m}{l}\right)^2$

145. Physical adsorption is weak.

146. Hot gases leave through chimnee.

147. 0.01 moles gives ----- x kJ
1 mole gives --------- ?
Then, heat of neutralisation = –100 x kj / mole

148. $3O_2$ + 69K cal $\rightleftarrows$ $2O_3$

149. Ligand is is different. 1, 3-prapanediamine and 1, 2-propanediamine

150. CH_2CHOH is glycolic acid

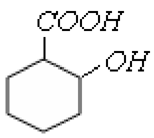
CH_3CHOH COOH is lactic acid

151. Hydrogen bonds

152. Asprin

153. A = C_2H_5Cl

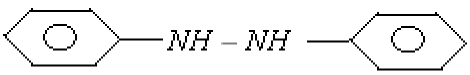
Ethyl chloride is not used to detect primary amines (Chloroforms is used)

154. $A =$ 

principle functional group is – COOH

155. Compound which contains α - hydrogens undergoes aldol condensation

156. C_6H_5 – COOH

157. 

$\sigma = 27, \pi = 6$

158. C_2H_5OH oxidises to CH_3CHO

CH_3CHO is substituted to give CCl_3CHO

159. Cane sugar is not reducing as anomenic carbon atoms can not involve in isomerisation.

160. Buffer capacity = $\frac{\text{number of moles of } OH^- \text{ added per litre}}{\text{change in pH}}$

$0.2 = \frac{8/40}{\text{change in pH}}$