Instructions :

i)   Each question carries one mark.

ii)   Choose the correct or most appropriate answer from the given options to the following questions and darken, with HB pencil, the corresponding digit 1,2,3 or 4 in the circle pertaining to the question number concerned in the OMR Answer Sheet, separately supplied to you.

MATHS

1. The distance between the parallel lilnes given by $(x+7y)^2 + 4\sqrt{2}(x+7y) - 42 = 0$ is

   1) 4/5   2) $4\sqrt{2}$ 3) 2   4) $10\sqrt{2}$

Sol. Key (3)

$$2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{8+42}{1(1+49)}} = 2$$

2. A point moves in the xy-plane such that the sum of its distances from two mutually perpendicular lines is always equal to 5 units. The area (in square units) enclosed by the locus of the point, is

   1) $\frac{25}{4}$ 2) 25   3) 50   4) 100

Sol. Key (3)

$$|x| + |y| = 5$$

$$\text{Area} = \frac{2c^2}{|ab|} = 50$$

3. The equation of a straight line passing through the point $(1, 2)$ and inclined at 45^0 to the line $y = 2x + 1$ is

   1) $5x + y = 7$ 2) $3x + y = 5$ 3) $x + y = 3$ 4) $x - y + 1 = 0$

Sol. Key (2)

$$\frac{m-2}{1+2m} = 1 \Rightarrow m - 2 = 1 + 2m$$

$$m = -3$$

$$y - 2 = -3(x-1) \Rightarrow 3x + y - 5 = 0$$

4. If a, b, c from a geometric progression with common ratio r, then the sum of the ordinates of the points of intersection of the line $ax + by + c = 0$ and the curve $x + 2y^2 = 0$ is

   1) $\frac{r^2}{2}$ 2) $-\frac{r}{2}$ 3) $\frac{r}{2}$ 4) r

Sol. Key (3)

$$a = a, b = ar, c = ar^2$$

$$ax + ary + ar^2 = 0$$

$$x + ry + r^2 = 0$$

$$-2y^2 + ry + r^2 = 0$$

$$\text{Sum} = +\frac{r}{2}$$

5. The point $(3, 2)$ undergoes the following three transformations in the order given
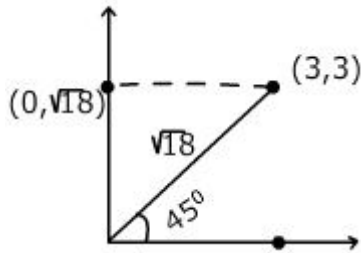
   (i) Reflection about the line $y = x$

   (ii) Translation by the distance 1 unit in the positive direction of x-axis

   (iii) Rotation by an angle $\dfrac{\pi}{4}$ about the origin in the anticlockwise direction

   Then the final position of the point is

   1) $\left(-\sqrt{18}, \sqrt{18}\right)$ 2) $(-2, 3)$ 3) $\left(0, \sqrt{18}\right)$ 4) $(0, 3)$

Sol. Key (3)

6. If X is a Poisson variate such that $\alpha = P(X = 1) = P(X = 2)$ then $P(X = 4) =$

   1) 2α 2) $\dfrac{\alpha}{3}$ 3) αe^{-2} 4) αe^{2}

Sol. Key (2)

$\alpha = P(X = 1) = P(X = 2)$

$\lambda = 2 \Rightarrow \alpha = 2e^{-2}$

$P(X = 4) = e^{-2}\dfrac{2^4}{4!} = \dfrac{2}{3}e^{-2} = \dfrac{\alpha}{3}$

7. Suppose X follows a binomial distribution with parameters n and p, where $0 < p < 1$. If $\dfrac{P(X = r)}{P(X = n - r)}$

   is independent of n for every r, then $p =$

   1) $\dfrac{1}{2}$ 2) $\dfrac{1}{3}$ 3) $\dfrac{1}{4}$ 4) $\dfrac{1}{8}$

Sol. Key (1)

$\dfrac{{}^{n}C_{r}\, p^{r}\, q^{n-r}}{{}^{n}C_{r-r}\, p^{n-r}q^{r}}$ independent of n

$\Rightarrow \left(\dfrac{p}{q}\right)^{2r-n} =$ Independent of n

$\Rightarrow \dfrac{p}{q} = 1 \Rightarrow p = q = \dfrac{1}{2}$

8. In an entrance test there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 9/10. If he gets the correct answer to a question, then the probability that he was guessing is

1) $\dfrac{37}{40}$ 2) $\dfrac{1}{37}$ 3) $\dfrac{36}{37}$ 4) $\dfrac{1}{9}$

Sol. Key (2)

$$\text{Required probability} = \dfrac{\dfrac{1}{10} \times \dfrac{1}{4}}{\dfrac{1}{10} \times \dfrac{1}{4} + \dfrac{9}{10} \times 1} = \dfrac{1}{37}$$

9. There are four machines and it is known that exactly two of them are faulty. They are tested one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

1) $\dfrac{1}{3}$ 2) $\dfrac{1}{6}$ 3) $\dfrac{1}{2}$ 4) $\dfrac{1}{4}$

Sol. Key (1)

$$\text{Required probability} = \dfrac{2}{4} \times \dfrac{1}{3} + \dfrac{2}{4} \times \dfrac{1}{3} = \dfrac{1}{3}$$

10. A fair coin is tossed 100 times. The probability of getting tails on odd number of times is

1) $\dfrac{1}{2}$ 2) $\dfrac{1}{4}$ 3) $\dfrac{1}{8}$ 4) $\dfrac{3}{8}$

Sol. Key (1)

$$\text{Odd number of times} = \dfrac{{}^{100}C_1 + {}^{100}C_3 + \ldots\ldots}{2^{100}}$$

$$= \dfrac{1}{2}$$

11. $\bar{a} = \bar{i} + \bar{j} - 2\bar{k} \Rightarrow \sum \left\{ \left(\bar{a} \times \bar{i}\right) \times \bar{j}\right\}^2 =$

1) $\sqrt{6}$ 2) 6   3) 36   4) $6\sqrt{6}$

Sol. Key (2)

$$\sum \left\{ \left(\bar{a} \times \hat{i}\right) \times \hat{j}\right\}^2 = \sum \left(x_2 \hat{j}\right)^2 = x_1^2 + x_2^2 + x_3^2 = 1 + 1 + 4 = 6$$

12. Let $\bar{a}$, $\bar{b}$ and $\bar{c}$ be three non-coplanar vectors and let $\bar{p}$, $\bar{q}$ and $\bar{r}$ be the vectors defined by

$$\bar{p} = \dfrac{\bar{b} \times \bar{c}}{\left[\bar{a}\bar{b}\bar{c}\right]}, \bar{q} = \dfrac{\bar{c} \times \bar{a}}{\left[\bar{a}\bar{b}\bar{c}\right]}, \bar{r} = \dfrac{\bar{a} \times \bar{b}}{\left[\bar{a}\bar{b}\bar{c}\right]}.$$

Then $\left(\bar{a} + \bar{b}\right).\bar{p} + \left(\bar{b} + \bar{c}\right).\bar{q} + \left(\bar{c} + \bar{a}\right).\bar{r} =$

1) 0   2) 1   3) 2   4) 3

Sol. Key (4)

$$\left(\bar{a} + \bar{b}\right).\bar{p} + \left(\bar{b} + \bar{c}\right).\bar{q} + \left(\bar{c} + \bar{a}\right).\bar{r}$$

$$= 1 + 1 + 1 = 3$$

13. Let $\bar{a} = \bar{i} + 3\bar{j} + \bar{k}$, $\bar{b} = \bar{i} - \bar{j} + \bar{k}$ and $\bar{c} = \bar{i} + \bar{j} - \bar{k}$. A vector in the plane of $\bar{a}$ and $\bar{b}$ has projection $\dfrac{1}{\sqrt{3}}$ on $\bar{c}$. Then, one such vector is

1) $4\bar{i} + \bar{j} - 4\bar{k}$ 2) $3\bar{i} + \bar{j} - 3\bar{k}$ 3) $4\bar{i} - \bar{j} + 4\bar{k}$ 4) $2\bar{i} + \bar{j} - 2\bar{k}$

Sol. Key (3)

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0 \Rightarrow x = z$

$\bar{r} = x\hat{i} + y\hat{j} + x\hat{k}$

$\dfrac{\bar{r}.\bar{c}}{|\bar{c}|} = \dfrac{1}{\sqrt{3}} = \dfrac{x + y - x}{\sqrt{x}} = \dfrac{1}{\sqrt{3}}$

$\bar{r} = 4\hat{i} - \hat{j} + 4\hat{k} \Rightarrow y = \pm 1$

14. The point of intersection of the lines

$l_1 = \bar{r}(t) = (\bar{i} - 6\bar{j} + 2\bar{k}) + t(\bar{i} + 2\bar{j} + \bar{k})$

$l_2 = \bar{R}(u) = (4\bar{j} + \bar{k}) + u(2\bar{i} + \bar{j} + 2\bar{k})$ is

1) $(4, 4, 5)$ 2) $(6, 4, 7)$ 3) $(8, 8, 9)$ 4) $(10, 12, 11)$

Sol. Key (3)

Equating the coefficients

$1 + t = 24 \Rightarrow 24 - t - 1 = 0$

$-6 + 2t = 4 + u \Rightarrow 4 - 2t + 10 = 0 \Rightarrow 2u - 4t + 20 = 0$

$t = 7$

$(8, 8, 9)$

15. The vectors $\overline{AB} = 3\bar{i} - 2\bar{j} + 2\bar{k}$ and $\overline{BC} = -\bar{i} - 2\bar{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is

1) $\dfrac{\pi}{2}$ 2) $\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$ 3) $\dfrac{3\pi}{4}$ or $\dfrac{\pi}{4}$ 4) $\dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$

Sol. Key (3)

$\overline{AC} = \overline{AB} + \overline{BC} = 2\hat{i} - 2\hat{j}$

$\overline{BD} = \overline{BC} - \overline{AB} = -4\hat{i} + 2\hat{j} - 4\hat{k}$

$\cos\theta = \dfrac{|-8 - 4|}{\sqrt{8 \times 6}} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

16. If p^{th}, q^{th}, r^{th} terms of a geometric progression are the positive numbers a, b, c respectively, then the angle between the vectors $\left(\log a^2\right)\bar{i} + \left(\log b^2\right)\bar{j} + \left(\log c^2\right)\bar{k}$ and $(q-r)\bar{i} + (r-p)\bar{j} + (p-q)\bar{k}$ is

1) $\dfrac{\pi}{3}$ 2) $\dfrac{\pi}{2}$ 3) $\sin^{-1}\dfrac{1}{\sqrt{a^2+b^2+c^2}}$ 4) $\dfrac{\pi}{4}$

Sol. Key (2)

$2\log a(q-r) + 2\log b(r-p) + 2\log c(p-q) = 0$

Angle $= \dfrac{\pi}{2}$

17. A vertical pole subtends an angle $\tan^{-1}\left(\dfrac{1}{2}\right)$ at a point P on the ground. If the angles subtended by the upper half and the lower half of the pole at P are respectively α and β, then $(\tan\alpha, \tan\beta) =$

1) $\left(\dfrac{1}{4}, \dfrac{1}{5}\right)$ 2) $\left(\dfrac{1}{5}, \dfrac{2}{9}\right)$ 3) $\left(\dfrac{2}{9}, \dfrac{1}{4}\right)$ 4) $\left(\dfrac{1}{4}, \dfrac{2}{9}\right)$
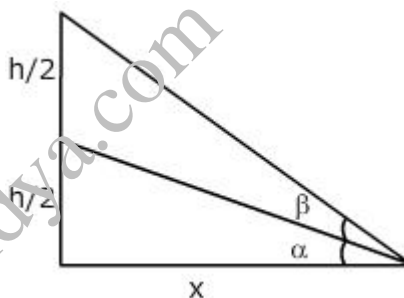
Sol. Key (3)

$\tan(\alpha+\beta) = \dfrac{1}{2} = \dfrac{h}{x} \Rightarrow x = 2h$

$\tan\alpha = \dfrac{h}{2x} = \dfrac{1}{4}$

$\tan\beta = \tan(\alpha+\beta-\alpha)$

$= \dfrac{\dfrac{1}{2} - \dfrac{1}{4}}{1 + \dfrac{1}{8}} = \dfrac{\dfrac{1}{4}}{\dfrac{9}{8}} = \dfrac{2}{9}$

$(\tan\alpha, \tan\beta) = \left(\dfrac{2}{9}, \dfrac{1}{4}\right)$

18. If α, β, γ are lengths of the altitudes of a triangle ABC with area Δ, then

1) $\sin^2 A + \sin^2 B + \sin^2 C$ 2) $\cos^2 A + \cos^2 B + \cos^2 C$

3) $\tan^2 A + \tan^2 B + \tan^2 C$ 4) $\cot^2 A + \cot^2 B + \cot^2 C$

Sol. Key (1)

$\alpha = \dfrac{2\Delta}{a}, \beta = \dfrac{2\Delta}{b}, \gamma = \dfrac{2\Delta}{c}$

$\dfrac{\Delta^2}{R^2}\left(\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2}\right) = \dfrac{\Delta^2}{R^2}\left[\dfrac{a^2+b^2+c^2}{4\Delta^2}\right] = \sin^2 A + \sin^2 B + \sin^2 C$

19. In an acute-angled triangle, $\cot B\cot C + \cot A\cot C + \cot A\cot B =$

1) -1   2) 0   3) 1   4) 2

Sol. Key (3)

In any triangle $\Sigma \cot A\cot B = 1$

20. $x = \log\left(\dfrac{1}{y} + \sqrt{1 + \dfrac{1}{y^2}}\right) \Rightarrow y =$

1) $\tanh x$ 2) $\coth x$ 3) $\sec hx$ 4) $\cos echx$

Sol. Key (4)

$$x = \log\left[\frac{1 + \sqrt{y^2 + 1}}{y}\right] = \cos ech^{-1} y \Rightarrow y = \cos echx$$

21. If $\dfrac{1}{2} \le x \le 1$ then $\cos^{-1} x + \cos^{-1}\left(\dfrac{x}{2} + \dfrac{1}{2}\sqrt{3 - 3x^2}\right) =$

1) $\dfrac{\pi}{6}$ 2) $\dfrac{\pi}{3}$ 3) π 4) 0

Sol. Key (2)

$$\frac{1}{2} \le x \le 1$$

Put $x = 1$

$$\cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right) = \frac{\pi}{3}$$

22. $3\sin x + 4\cos x = 5 \Rightarrow 6\tan\dfrac{x}{2} - 9\tan^2\dfrac{x}{2} =$

1) 0   2) 1   3) 3   4) 4

Sol. Key (2)

$$3\sin x + 4\cos x = 5$$

$$3\left(\frac{2t}{1 + t^2}\right) + 4\left(\frac{1 - t^2}{1 + t^2}\right) = 5$$

$$6t + 4 - 4t^2 = 5 + 5t^2$$

$$9t^2 - 6t + 1 = 0 \Rightarrow (3t - 1)^2 = 1$$

$$t = \frac{1}{3}$$

$$6\left(\frac{1}{3}\right) - 9\left(\frac{1}{9}\right) = 2 - 1 = 1$$

23. $\tan x + \tan\left(x + \dfrac{\pi}{3}\right) + \tan\left(x + \dfrac{2\pi}{3}\right) = 3 \Rightarrow \tan 3x =$

1) 3   2) 2   3) 1   4) 0

Sol. Key (3)

$$\tan x + \tan\left(x = \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$3\tan 3x = 3 \Rightarrow \tan 3x = 1$$

24. $\cos 36^0 - \cos 72^0 =$

1) 1   2) $\dfrac{1}{2}$ 3) $\dfrac{1}{4}$ 4) $\dfrac{1}{8}$

Sol. Key (2)

$$\cos 36^0 - \cos 72^0 = \frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4} = \frac{1}{2}$$

25. The minimum value of $27\tan^2\theta + 3\cot^2\theta$ is

1) 15   2) 18   3) 24   4) 30

Sol. Key (2)

$2\sqrt{27 \times 3} = 18$

26. If α is a non real root of the equation $x^6 - 1 = 0$ then $\dfrac{\alpha^2 + \alpha^3 + \alpha^4 + \alpha^5}{\alpha + 1} =$

1) α 2) 1   3) 0   4) -1

Sol. Key (4)

$$\frac{\alpha^2 + \alpha^3 + \alpha^4 + \alpha^5}{\alpha + 1} = \frac{-(\alpha + 1)}{\alpha + 1} = -1$$

27. If $f : R \to R^+$ and $g : R^+ \to R$ are such that $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then a possible choice for f and g is

1) $f(x) = x^2, g(x) = \sin\sqrt{x}$ 2) $f(x) = \sin x, g(x) = |x|$

3) $f(x) = \sin^2 x, g(x) = \sqrt{x}$ 4) $f(x) = x^2, g(x) = \sqrt{x}$

Sol. Key (3)

By verification

28. If $f : Z \to Z$ is defined by $f(x) = \begin{cases} \dfrac{x}{2} & \text{if x is even} \\ 0 & \text{if x is odd} \end{cases}$ then f is

1) onto but not one to one   2) one to one but not onto

3) one to one and onto   4) neither one to one nor onto

Sol. Key (1)

Onto but not one to one

29. If $\dfrac{1}{2 \times 4} + \dfrac{1}{4 \times 6} + \dfrac{1}{6 \times 8} + \ldots\ldots(n - \text{terms}) = \dfrac{kn}{n+1}$ then $k =$

1) $\dfrac{1}{4}$ 2) $\dfrac{1}{2}$ 3) 1   4) $\dfrac{1}{8}$

Sol. Key (1)

Put $n = 1$

$$\frac{1}{8} = \frac{k}{2} \Rightarrow k = \frac{1}{4}$$

30. A regular polygon of n sides has 170 diagonals. Then $n =$

1) 12   2) 17   3) 20   4) 25

Sol. Key (3)

$$\frac{n(n-3)}{2} = 170 \Rightarrow n = 20$$

31. A committee of 12 members is to be formed from 9 women and 8 men. The number of committees in which the women are in majority is

1) 2720   2) 2702   3) 2270   4) 2278

Sol. Key (2)

W	M	
9	8	12
7	5	
8	4	
9	3	

Required $= {}^9C_7\,{}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3 = 2702$.

32. A student has to answer 10 out of 13 questions in an examination choosing at least 5 questions from the first 6 questions. The number of choices available to the student is

1) 63   2) 91   3) 161   4) 196

Sol. Key (3)

6	7
5	5
6	4

Required $= {}^6C_5 \times {}^7C_5 + {}^6C_6 + {}^7C_4 = 161$.

33. $\displaystyle \sum_{k=1}^{\infty} \sum_{r=0}^{k} \frac{1}{3^k} \left({}^kC_r \right) =$

1) $\dfrac{1}{3}$ 2) $\dfrac{2}{3}$ 3) 1   4) 2

Sol. Key (4)

$$\sum_{k=1}^{\infty} \left(\sum_{r=0}^{k} \frac{{}^kC_r}{3^k} \right) = \sum_{k=1}^{\infty} \left(\frac{2}{3} \right)^k = 2$$

34. If $ab \neq 0$ and the sum of the coefficients of x^7 and x^4 in the expansion of $\left(\dfrac{x^2}{a} - \dfrac{b}{x} \right)^{11}$ is zero, then

1) $a = 0$ 2) $a + b = 0$ 3) $ab = -1$ 4) $ab = 1$

Sol. Key (4)

$ab = 1$

35. $\dfrac{1}{x(x+1)(x+2)....(x+n)} = \dfrac{A_0}{x} + \dfrac{A_1}{x+1} + \dfrac{A_2}{x+2} + ... + \dfrac{A_n}{x+n}$, $0 \leq r \leq n \Rightarrow A_r =$

1) $(-1)^r \dfrac{r^1}{(n-r)!}$ 2) $(-1)^r \dfrac{1}{r!(n-r)!}$ 3) $\dfrac{1}{r!(n-r)!}$ 4) $\dfrac{r!}{(n-r)!}$

Sol. Key (2)

Put $x = -r$

$$A_r = \frac{(-1)^r}{r!(n-r)!}$$

36. $1 + \dfrac{1}{3.2^2} + \dfrac{1}{5.2^4} + \dfrac{1}{7.2^6} + ... =$

   1) $\log_e 2$ 2) $\log_e 3$ 3) $\log_e 4$ 4) $\log_e 5$

Sol. Key (2)

$$\left[\log\left(\dfrac{1 + \dfrac{1}{2}}{1 - \dfrac{1}{2}} \right) \right] = \log_e 3$$

37. In a triangle PQR, $\angle R = \dfrac{\pi}{4}$. If $\tan\left(\dfrac{P}{3}\right)$ and $\tan\left(\dfrac{Q}{3}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then

   1) $a + b = c$ 2) $b + c = 0$ 3) $a + c = b$ 4) $b = c$

Sol. Key (1)

$$\angle R = \dfrac{\pi}{4} \Rightarrow \angle P + \angle Q = \dfrac{3\pi}{4} \Rightarrow \dfrac{P}{3} + \dfrac{Q}{3} = \dfrac{\pi}{4}$$

$$\tan\dfrac{P}{3} + \tan\dfrac{Q}{3} = 1 - \tan\dfrac{P}{3}\tan\dfrac{Q}{3}$$

$$a + b = c$$

38. The product of real roots of the equation $|x|^{\frac{6}{5}} - 26|x|^{\frac{3}{5}} - 27 = 0$ is

   1) -3^{10} 2) -3^{12} 3) $-3^{\frac{12}{5}}$ 4) $-3^{\frac{21}{5}}$

Sol. Key (1)

Put $|x|^{3/5} = t$

$t^2 - 26t - 27 = 0$

$(t - 27)(t + 1) = 0$

$t = 27$

$|x|^{3/5} = 27$, $|x| = 3^5$

$x = \pm 3^5$

Product $= -3^{10}$

39. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then the coefficient of x in the cubic equation whose roots are $\alpha(\beta + \gamma), \beta(\gamma + \alpha)$ and $\gamma(\alpha + \beta)$ is

   1) $2q$ 2) $q^2 + pr$ 3) $p^2 - qr$ 4) $r(pq - r)$

Sol. Key (2)

Put $\alpha = 1, \beta = 2, \gamma = 3$

$q^2 + pr$

40. Let $A = \begin{vmatrix} 2 & e^{i\pi} \\ i^2 & i^{2012} \end{vmatrix}$, $C = \dfrac{d}{dx}\left(\dfrac{1}{x}\right)\bigg|_{x=1}$ and $D = \displaystyle\int_{e^2}^{1} \dfrac{dx}{x}$. If the sum of two roots of the equation

$Ax^3 + Bx^2 + Cx + D = 0$ is equal to zero, then B =

1) -1   2) 0   3) 1   4) 2

Sol. Key (4)

$A = 1, C = -1, D = -2$

$x^3 + Bx^2 - x - 2 = 0$ $\alpha + \beta + \gamma = -B$

$-B^3 + B^3 + B - 2 = 0$ $\gamma = -B$

$B = 2$

41. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow A^8 =$

1) 4B   2) 8B   3) 64B   4) 128B

Sol. Key (3)

$A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = -2B$

$A^8 = 2^4 B^4 = 16 \times 4B = 64B.$

42. $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x-1)x(x+1) \end{vmatrix} \Rightarrow f(2012) =$

1) 0   2) 1   3) –500   4) 500

Sol. Key (1)

$f(x) = 0$

$f(2012)\ 0.$

43. Let $A = \begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$ If a, b and c respectively denote the ranks of

A, B and C then the correct order of these numbers is

1) a < b < c   2) c < b < a   3) b < a < c   4) a < c < b

Sol. Key (3)

$P(A) = 2\ P(B) = 1\ P(C) = 3$

$b < a < c$

44. Given that $a\alpha^2 + 2b\alpha + c \neq 0$ and that the system of equations

$(a\alpha + b)x + ay + bz = 0$

$(b\alpha + c)x + by + cz = 0$

$(a\alpha + b)y + (b\alpha + c)z = 0$

has a non-trivial solution, then a, b, c lie in

1) Arithmetic progression   2) Geometric progression

3) Harmonic progression   4) Arithmetico-geometric progression

Sol. Key (2)

$$\begin{vmatrix} a\alpha + b & a & b \\ b\alpha + c & b & c \\ 0 & a\alpha + b & b\alpha + c \end{vmatrix} = 0 \quad C_1 \rightarrow C_1 - \alpha C_2 - C_3$$

$$\begin{vmatrix} 0 & a & b \\ 0 & b & c \\ -\left(a\alpha^2 + 2b\alpha + c\right) & a\alpha + b & b\alpha + c \end{vmatrix} = 0$$

$$\left(a\alpha^2 + 2b\alpha + c\right)\left(ac - b^2\right) = 0$$

a, b, c G.P.

45. If a, b, c, d $\in$ R are such that $a^2 + b^2 = 4$ and $c^2 + d^2 = 2$ and if $(a + ib)^2 = (c = id)^2 (x + iy)$ then $x^2 + y^2 =$

1) 4   2) 3   3) 2   4) 1

Sol. Key (3)

$$\left(\frac{a + ib}{c + id}\right)^2 = x + iy$$

Take modulus on both sides

$$\frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2 = \frac{4}{2} = 2$$

46. If z is a complex number such that $\left|z - \dfrac{4}{z}\right| = 2$, then the greatest value of |z| is

1) $1 + \sqrt{2}$ 2) $\sqrt{2}$ 3) $\sqrt{3} + 1$ 4) $1 + \sqrt{5}$

Sol. Key (4)

$$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \le \left|z - \frac{4}{z}\right| + \frac{4}{|z|} = 2 + \frac{4}{|z|}$$

$$|z|^2 - 2|z| - 4 \le 0$$

$$\Rightarrow |z| = 1 + \sqrt{5}.$$

47. An integrating factor of the differential equation $\left(1 - x^2\right)\dfrac{dy}{dx} + xy = \dfrac{x^4}{\left(1 + x^5\right)}\left(\sqrt{1 - x^2}\right)^3$ is

1) $\sqrt{1 - x^2}$ 2) $\dfrac{x}{\sqrt{1 - x^2}}$ 3) $\dfrac{x^2}{\sqrt{1 - x^2}}$ 4) $\dfrac{1}{\sqrt{1 - x^2}}$

Sol. Key (4)

$$\frac{dy}{dx} + \frac{x}{1 - x^2}y = \frac{x^4}{1 + x^5}\left(1 - x^2\right)^{5/2}$$

$$\text{I.F.} = e^{\int \frac{x}{1 - x^2}} = \frac{1}{\sqrt{1 - x^2}}$$

48. $\dfrac{dy}{dx} + 2x \tan(x-y) = 1 \Rightarrow \sin(x-y) =$

1) Ae^{-x^2} 2) Ae^{2x} 3) Ae^{x^2} 4) Ae^{-2x}

Sol. Key (1)

Put $x - y = z$

$\dfrac{dz}{dx} + 2x \tan z = 0$

$\log|\sin z| + x^2 = A$

$\sin(x-y) = Ae^{-x^2}$

49. The value of the integral $\displaystyle\int_0^4 \dfrac{dx}{1+x^2}$ obtained by using Trapezoidal rule with h = 1 is

1) $\dfrac{63}{85}$ 2) $\tan^{-1}(4)$ 3) $\dfrac{108}{85}$ 4) $\dfrac{113}{85}$

Sol. Key (4)

$\displaystyle\int_0^4 \dfrac{dx}{1+x^2} = \dfrac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right] = \dfrac{113}{85}$

50. The area (in square units) bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is

1) $\dfrac{64}{3}$ 2) $\dfrac{16}{3}$ 3) $\dfrac{8}{3}$ 4) $\dfrac{2}{3}$

Sol. Key (2)

$\dfrac{16ab}{3} = \dfrac{16 \times 1 \times 1}{3} = \dfrac{16}{3}$

51. $a > 0, \displaystyle\int_{-\pi}^{\pi} \dfrac{\sin^2 x}{1+a^x} dx =$

1) $\dfrac{\pi}{2}$ 2) π 3) $\dfrac{a\pi}{2}$ 4) $a\pi$

Sol. Key (1)

Put $x = -t$

$2I = \displaystyle\int_{-\pi}^{\pi} \sin^2 x \Rightarrow I = \dfrac{\pi}{2}$

52. $\displaystyle\int \dfrac{dx}{\sqrt{x - x^2}} =$

1) $2\sin^{-1}\sqrt{x} + c$ 2) $2\sin^{-1}x + c$ 3) $2x\sin^{-1}x + c$ 4) $\sin^{-1}\sqrt{x} + c$

Sol. Key (1)

$\displaystyle\int \dfrac{dx}{\sqrt{x - x^2}} = 2\sin^{-1}\sqrt{x} + c$

53. $\int \sec^2 x \, \text{cosec}^4 x \, dx = -\dfrac{1}{3} \cot^3 x + k \tan x - 2 \cot x + c \Rightarrow k =$

   1) 4   2) 3   3) 2   4) 1

Sol. Key (4)

$$\int \sec^2 x \cos ec^4 x \, dx = \int \frac{1}{\sin^4 x} dx$$

$$= \int \left(\sec^2 x + \cos ec^2 x + \cos ec^4 x \right) dx$$

$$= \tan x - \cot x - \frac{\cot^3 x}{3} - x + c$$

$$k = 1$$

54. $\int \dfrac{dx}{x^2 \sqrt{4 + x^2}} =$

   1) $\dfrac{1}{4}\sqrt{4 + x^2} + c$ 2) $\dfrac{-1}{4}\sqrt{4 + x^2} + c$ 3) $\dfrac{-1}{4x}\sqrt{4 + x^2} + c$ 4) $\dfrac{9}{4x}\sqrt{4 + x^2} + c$

Sol. Key (3)

$$\int \frac{dx}{x^2 \sqrt{4 + x^2}} = \int \frac{dx}{x^3 \left(\frac{4}{x^2} + 1 \right)^{1/2}}$$

$$= -\frac{1}{4x}\sqrt{4 + x^2} + c$$

55. If $u = f(r)$, where $r^2 = x^2 + y^2$ then $\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} \right) =$

   1) $f''(r)$ 2) $f''(r) + f'(r)$ 3) $f''(r) + \dfrac{1}{r} f'(r)$ 4) $f''(r) + r f'(r)$

Sol. Key (3)

Formula $f^{11}(r) + \dfrac{1}{r} f^1(r)$

56. If the volume of a sphere increases at the rate of 2π cm^3/sec, then the rate of increase of its radius (in cm/sec), when the volume is 228π cm^3 is

   1) $\dfrac{1}{36}$ 2) $\dfrac{1}{72}$ 3) $\dfrac{1}{18}$ 4) $\dfrac{1}{9}$

Sol. Key (2)

$$\frac{dV}{dt} = 2\pi$$

$$V = 288\pi \Rightarrow r = 6$$

$$\frac{dr}{dt} = \frac{1}{72}$$

57. If Δ is the area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, then $\Delta =$

   1) $\dfrac{\sqrt{3}}{2}$ 2) $\sqrt{3}$ 3) $2\sqrt{3}$ 4) 6

Sol. Key (3)

$$\text{Area} = \frac{y_1^2(1 + m^2)}{2|m|}$$

$$m = -\left[\frac{x}{y}\right] = -\sqrt{3}$$

$$\frac{3\left(\dfrac{4}{3}\right)}{\dfrac{2}{\sqrt{3}}} = 2\sqrt{3}$$

58. The coordinates of the point P on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ where the tangent is inclined at the angle $\dfrac{\pi}{4}$ to the x-axis, are

   1) $\left(a\left(\dfrac{\pi}{2} - 1\right), a\right)$ 2) $\left(a\left(\dfrac{\pi}{2} + 1\right), a\right)$ 3) $\left(a\dfrac{\pi}{2}, a\right)$ 4) (a, a)

Sol. Key (2)

$$\frac{dy}{dx} = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$P = \left(a\left(\frac{\pi}{2} + 1\right), a\right)$$

59. $f(x) = (x^2 - 1)^7 \Rightarrow f^{(14)}(x)$

   1) 0 2) 2! 3) 7! 4) 14!

Sol. Key (4)

$$f^{(14)}(x) = 14!$$

60. $x^2 + y^2 = t + \dfrac{1}{t}$, $x^4 + y^4 = t^2 + \dfrac{1}{t^2} \Rightarrow x^3 y \dfrac{dy}{dx} =$

   1) -1 2) 1 3) 0 4) t

Sol. Key (1)

$$x^4 + y^4 = \left(x^2 + y^2\right)^2 - 2x^2y^2$$

$$t^2 + \frac{1}{t^2} = \left(t + \frac{1}{t}\right)^2 - 2x^2y^2 = t^2 + \frac{1}{t^2} + 2 - 2x^2y^2$$

$$x^2y^2 = 1 \Rightarrow y^2 = \frac{1}{x^2}$$

$$2y\frac{dy}{dx} = \frac{-2}{x^3}$$

$$x^3 y \frac{dy}{dx} = -1$$

61. If $xy \neq 0$, $x + y \neq 0$ and $x^m y^n = (x+y)^{m+n}$ where $m, n \in N$ then $\dfrac{dy}{dx} =$

   1) $\dfrac{y}{x}$ 2) $\dfrac{x+y}{xy}$ 3) xy 4) $\dfrac{x}{y}$

Sol. Key (1)

   for Homo function $\dfrac{dy}{dx} = \dfrac{y}{x}$.

62. $f(x) = \log\left(e^x \left(\dfrac{x-2}{x+2} \right)^{3/4} \right) \Rightarrow f'(0) =$

   1) $1/4$ 2) 4 3) $-3/4$ 4) 1

Sol. Key (1)

   $f(x) = x + \dfrac{3}{4} \log(x-2) - \dfrac{3}{4} \log(x+2)$

   $f'(x)\ 1 + \dfrac{3}{4(x-2)} - \dfrac{3}{4(x+2)}$

   $f'(0) = 1 - \dfrac{3}{8} - \dfrac{3}{8} = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

63. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} \alpha + \dfrac{\sin[x]}{x} & \text{if } x > 0 \\[2mm] 2 & \text{if } x = 0 \\[2mm] \beta + \left[\dfrac{\sin x - x}{x^3} \right] & \text{if } x < 0 \end{cases}$$

   where $[y]$ denotes the integral part of y,. If f is continuous at $x = 0$, then $\beta - \alpha =$

   1) -1 2) 1 3) 0 4) 2

Sol. Key (2)

   $LHL = RHL = f(0)$

   $\Rightarrow \alpha = 2, \beta = 3, \beta - \alpha = 1$.

64. $\displaystyle \lim_{x \to \infty} \left(\dfrac{x+6}{x+1} \right)^{x+4} =$

   1) e^4 2) e^6 3) e^5 4) e

Sol. Key (3)

   $e^{\lim(x+4)\left(\frac{5}{x+1} \right)} = e^5$.

65. The equation of the sphere through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, 1)$ and having the smallest radius is

1) $3(x^2 + y^2 + z^2) - 4x - 4y - 2z + 1 = 0$ 2) $2(x^2 + y^2 + z^2) - 3x - 3y - z + 1 = 0$

3) $x^2 + y^2 + z^2 - x - y + z + 1 = 0$ 4) $x^2 + y^2 + z^2 - 2x - 2y + 4z + 1 = 0$

Sol. Key (1)

by verification.

66. If the foot of the perpendicular from $(0, 0, 0)$ to a plane is $(1, 2, 3)$, then the equation of the plane is

1) $2x + y + 3z = 14$ 2) $x + 2y + 3z = 14$ 3) $x + 2y + 3z + 14 = 0$ 4) $x + 2y - 3z = 14$

Sol. Key (2)

$1(x - 1) + 2(y - 2) + 3(z - 3) = 0$

$x + 2y + 3z = 14$.

67. A straight line is equally inclined to all the three coordinate axes. Then an angle made by the line with the y-axis is

1) $\cos^{-1}\left(\dfrac{1}{3}\right)$ 2) $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ 3) $\cos^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ 4) $\dfrac{\pi}{4}$

Sol. Key (2)

$l = m = n = \dfrac{1}{\sqrt{3}} \quad \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$.

68. If x-coordinate of a point P on the line joining the points $Q(2, 2, 1)$ and $R(5, 1, -2)$ is 4, then the z-coordinate of P is

1) -2 2) -1 3) 1 4) 2

Sol. Key (2)

$\dfrac{x - 2}{2 - 5} = \dfrac{z - 1}{1 + 2} \Rightarrow \dfrac{4 - 2}{-3} = \dfrac{z - 1}{3} \Rightarrow z - 1 = -2$

$z = -1$.

69. The radius of the circle $r = 12 \cos \theta + 5 \sin \theta$ is

1) $\dfrac{5}{12}$ 2) $\dfrac{17}{2}$ 3) $\dfrac{15}{2}$ 4) $\dfrac{13}{2}$

Sol. Key (4)

$x^2 + y^2 - 12x - 5y = 0$

$r = \sqrt{36 + \dfrac{25}{4}} = \dfrac{13}{2}$.

70. The area (in square units) of the equilateral triangle formed by the tangent at $\left(\sqrt{3}, 0\right)$ to the hyperbola $x^2 - 3y^2 = 3$ with the pair of asymptotes of the hyperbola is

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\dfrac{1}{\sqrt{3}}$ 4) $2\sqrt{3}$

Sol. Key (2)

$x - \sqrt{3}y = 0 \quad \left(\sqrt{3}, 1\right)$

$x + \sqrt{3}y = 0 \quad \left(\sqrt{3}, -1\right)$

$x = \sqrt{3} \quad (0, 0)$

$A = \sqrt{3}$.

71. Equation of one of the tangents passing through (2, 8) to the hyperbola $5x^2 - y^2 = 5$ is
   1) $3x + y - 14 = 0$ 2) $3x - y + 2 = 0$ 3) $x + y + 3 = 0$ 4) $x - y + 6 = 0$

Sol. Key (2)

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

(2, 8)

$3x - y + 2 = 0$.

72. If the line $2x + 5y = 12$ intersects the ellipse $4x^2 + 5y^2 = 20$ in two distinct points A and B, then the mid point of AB is
   1) (0, 1)   2) (1, 2)   3) (1, 0)   4) (2, 1)

Sol. Key (2)

$S_1 = S_{11}$

(1, 2).

73. Let $x + y = k$ be a normal to the parabola $y^2 = 12x$. If p is the length of the perpendicular from the focus of the parabola onto this normal, then $4k - 2p^2 =$
   1) 1   2) 0   3) –1   4) 2

Sol. Key (2)

$y = -x + k$ $S = (3, 0)$ $x + y - 9 = 0$

$k = -2am - am^3 = 9.$ $P = \dfrac{|6|}{\sqrt{2}} = 3\sqrt{2}$

$4k - 2p^2 = 36 - 36 = 0.$

74. The equation to the line joining the centres of the circles belonging to the coaxial system of circles $4x^2 + 4y^2 - 12x + 6y - 3 + \lambda(x + 2y - 6) = 0$ is
   1) $8x - 4y - 15 = 0$ 2) $8x - 4y + 15 = 0$ 3) $3x - 4y - 5 = 0$ 4) $3x - 4y + 5 = 0$

Sol. Key (1)

$$C = \left(\dfrac{3}{2}, \dfrac{-3}{4} \right)$$

$2x - y + k = 0$

$3 + \dfrac{3}{4} + k = 0 \Rightarrow k = \dfrac{-15}{4}$

$8x - 4y - 15 = 0.$

75. A circle passes through the point (3, 4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally; the locus of its centre is a straight line. If the distance of this straight line from the origin is 25, then $a^2 =$
   1) 250   2) 225   3) 100   4) 25

Sol. Key (2)

$$d^2 = r_1^2 + r_2^2$$

$$x_1^2 + y_1^2 = a^2 + (x_1 - 3)^2 + (y_1 - 4)^2$$

$$6x + 8y = a^2 + 25$$

$$\dfrac{a^2 + 25}{10} = 25 \Rightarrow a^2 = 225$$

76. If the line $x + 3y = 0$ is the tangent at $(0, 0)$ to circle of radius 1, then the centre of one such circle is

1) $(3, 0)$ 2) $\left(\dfrac{-1}{\sqrt{10}}, \dfrac{3}{\sqrt{10}}\right)$ 3) $\left(\dfrac{3}{\sqrt{10}}, \dfrac{-3}{\sqrt{10}}\right)$ 4) $\left(\dfrac{1}{\sqrt{10}}, \dfrac{3}{\sqrt{10}}\right)$

Sol. Key (4)

By verification

77. Consider the circle $x^2 + y^2 - 4x - 2y + c = 0$ whose centre is A(2, 1). If the point P(10, 7) is such that the line segment PA meets the circle in Q with PQ = 5, then c =

1) -15   2) 20   3) 30   4) -20

Sol. Key (4)

$A(2,1), P(10,7)$

$AP = 10$

$PQ = 5$

$r = 5$
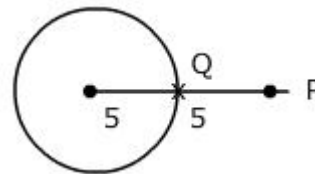
$\sqrt{4 + 1 - C} = 5 \Rightarrow 5 - C = 25 \Rightarrow C = -20$

78. Given the circle C with the equation $x^2 + y^2 - 2x + 10y - 38 = 0$.
Match the List – I with the List – II given below concerning C :

List – I	List – II
i) The equation of the polar of (4, 3) with respect to C	a) $y + 5 = 0$
ii) The equation of the tangent at (9, –5) on C	b) $x = 1$
iii) The equation of the normal at (–7, –5) on C	c) $3x + 8y = 27$
iv) The equation of the diameter of C passing through (1, 3)	d) $x + y = 3$
	e) $x = 9$

The correct answer is

	(i)	(ii)	(iii)	(iv)			(i)	(ii)	(iii)	(iv)
1)	c	a	e	b		2)	d	e	a	b
3)	c	e	a	b		4)	d	b	a	e

Sol. Key (3)

List - (i)

$I : S_1 = 0$

$II : S_1 = 0$

$III : CP$ eq.

$IV : C(1, -5) A(1, 3)$ eq.

79. If the pair of lines given by $(x^2 + y^2)\cos^2\theta = (x\cos\theta + y\sin\theta)^2$ are perpendicular to each other, then $\theta =$

1) 0   2) $\dfrac{\pi}{4}$ 3) $\dfrac{\pi}{3}$ 4) $3\dfrac{\pi}{4}$

Sol. Key (2)

x^2 coefficient $+ y^2$ coefficient $= 0$

$2\cos^2\theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

80. If the area of the triangle formed by the pair of lines $8x^2 - 6xy + y^2 = 0$ and the line $2x + 3y$ a is 7 then a=

1) 14   2) $14\sqrt{2}$ 3) $28\sqrt{2}$ 4) 28

Sol. Key (4)

$$\left| \frac{n^2 \sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2} \right| = 7 \Rightarrow a = 28$$

PHYSICS

81. The gravitational field in a region is given by equation $\overline{E} = \left(5\hat{i} + 12\hat{j}\right) N/kg$. If a particle of mass 2 kg is moved from the origin to the point (12 m, 5m) in this region, the change in gravitational potential energy is

1) -225 J   2) -240 J   3) -245 J   4) -250 J

Key (2)

Sol :- $\Delta v = U_2 - U_1 = -m \int\limits_{r_1}^{r_2} \vec{E}.\vec{dr}$

$$= -(2kg) \int \left(5\hat{i} + 12\hat{j}\right).\left(12\hat{i} + 5\hat{j}\right)$$

$$= -240 \, J$$

82. The time period of a particle in simple harmonic motion is 8s. At t = 0, it is at the mean position. The ratio of the distances travelled by it in the first and seconds is

1) $\dfrac{1}{2}$ 2) $\dfrac{1}{\sqrt{2}}$ 3) $\dfrac{1}{\sqrt{2}-1}$ 4) $\dfrac{1}{\sqrt{3}}$

Key (3)

Sol :- T = 8s

$x = A \, Sin \, wt$

$x_1 = A \, Sin\left(\dfrac{2\pi}{T}\right)(1)$

$x_1 = A \, Sin\left(\dfrac{2\pi}{8}\right)1$

$x_1 = \dfrac{A}{\sqrt{2}}$

$x_2 = A - \dfrac{A}{\sqrt{2}}$

$\therefore x_1 : x_2 = \dfrac{1}{\sqrt{2}-1}$

83. A tension of 22 N is applied to a copper wire of cross-sectional area 0.02 cm² Young's modulus of copper is 1.1 x 1011 N/m2 and Poisson's ratio 0.32. The decrease in cross sectional area will be

1) 1.28 x 10⁻⁶ cm²   2) 1.6 x 10⁻⁶ cm²   3) 2.56 x 10⁻⁶ cm²   4) 0.64 x 10⁻⁶ cm²

Key (1)

Sol :- $\sigma = \dfrac{\left(\dfrac{-\Delta r}{r}\right)}{\dfrac{\Delta L}{L}}$ $\qquad \therefore \dfrac{\Delta r}{r} = \sigma \left(\dfrac{\Delta L}{L}\right)$

$\therefore \dfrac{\Delta r}{r} = \sigma \left(\dfrac{F}{AY}\right)$

$\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r} = \dfrac{2\sigma F}{AY}$ $\qquad \therefore \Delta A = \dfrac{2\sigma F}{Y}$

$\therefore \Delta A = 1.28 \times 10^{-6} cm^2$

84. Drops of liquid of density 'd' are floating half immersed in a liquid of density ρ. If the surface tension of the liquid is T, then the radius of the drop is

1) $\sqrt{\dfrac{3T}{g(3d-\rho)}}$
   2) $\sqrt{\dfrac{6T}{g(2d-\rho)}}$
   3) $\sqrt{\dfrac{3T}{g(2d-\rho)}}$
   4) $\sqrt{\dfrac{3T}{g(4d-3\rho)}}$

Key (3)

Sol:- $\dfrac{V}{2}.\rho.g + (2\pi r)T = Vdg$

$r = \sqrt{\dfrac{3T}{g(2d-\rho)}}$ $\qquad \left(V = \dfrac{4}{3}\pi r^3\right)$

85. A pipe having an internal diameter 'D' is connected to another pipe of same size. Water flows into the second pipe through 'n' holes, each of diameter 'd'. If the water in the first pipe has speed 'v', the speed of water leaving the second pipe is

1) $\dfrac{D^2v}{nd^2}$
   2) $\dfrac{nD^2v}{d^2}$
   3) $\dfrac{nd^2v}{D^2}$
   4) $\dfrac{d^2v}{nD^2}$

Key (1)

Sol :- $\left(\dfrac{\pi}{4}D^2\right)V = n \times \left(\dfrac{\pi}{4}d^2\right) \times V'$

$V' = \dfrac{D^2V}{nd^2}$

86. When a liquid is heated in copper vessel its coefficient of apparent expansion is 6 x 10⁻⁶/⁰C. When the same liquid is heated in a steel vessel its coefficient of apparent expansion is 6 x 10⁻⁶/⁰C. If coefficient of linear expansion for copper is 18 x 10⁻⁶/⁰C, the coefficient of lineare expansion for steel is

1) 20 x 10⁻⁶/⁰C
   2) 24 x 10⁻⁶/⁰C
   3) 36 x 10⁻⁶/⁰C
   4) 12 x 10⁻⁶/⁰C

Key (4)

Sol :- $\gamma_{ac} + 3\alpha_c = \gamma_{as} + 3\alpha_s$

$\therefore \alpha_s = 12 \times 10^{-6} /^0 C$

87. When the temperature of a body increases from T to T + Δ T, its moment of inertia increases from I to I + Δ I. If α is the coefficient of linear expansion of the material of the body, then $\dfrac{\Delta I}{I}$ is (neglect higher orders of α)

1) $\alpha \Delta T$
   2) $2\alpha \Delta T$
   3) $\dfrac{\Delta T}{\alpha}$
   4) $\dfrac{2\alpha}{\Delta T}$

Key (2)

Sol :- $I = MK^2$

$$\frac{\Delta I}{I} = 2\frac{\Delta K}{K} = 2 \propto \Delta T$$

88. A sound wave passing through an ideal gas at NTP produces a pressure change of 0.001 dyne/cm² during adiabatic compression. The corresponding change in termperature

($\gamma = 1.5$ for the gas and atmospheric pressure is 1.013×10^6 dynes/cm²) is

1) 8.97×10^{-4} K   2) 8.97×10^{-6} K   3) 8.97×10^{-8} K   4) 8.97×10^{-9} K

Key (3)

Sol :- $T^r P^{1-r} = $ constant------------(1)

$or \quad T^r = KP^{1-r}$

Differentiations

$$\Delta T = \frac{(r-1)}{r}\left(\frac{T}{P}\right)\Delta P$$

Substitution values : $\Delta T = 8.97 \times 10^{-8} K$

89. Work done to increase the temperature of one mole of an ideal gas by 30°C, if it is expanding under the condition $V \propto T^{2/3}$ is, (R= 8.314 J/mole/°K)

1) 116.2 J   2) 136.2 J   3) 166.2 J   4) 186.2 J

Key (3)

Sol :- $W = \left(\frac{nR}{x-1}\right)\Delta T \quad \left[PV^{-1/2} = cons\tan t\right]$

n = 1 mole   x = -1/2

R = 8.314 J/mole K

$\Delta T = 30^0 C$

$\therefore W = 166.2 J$

90. Power radiated by a black body at temperature T_1 is P and it radiates maximum energy at a wavelength λ_1. If the temperature of the black body is changed from T_1 to T_2, it radiates maximum energy at a wavelength $\frac{\lambda_1}{2}$. The power radiated at T_2 is

1) 2P   2) 4P   3) 8P   4) 16P

Key (4)

Sol :- $\frac{W_1}{f_1} + \frac{W_2}{f_2} = 0$

$P \propto T_1^4 \quad\quad T_1\lambda_1 = T_2\left(\frac{\lambda_1 2}{}\right)$

$\therefore T_2 = 2T_1$

$P' \propto T_2^4 \Rightarrow P' = 16P$

91. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a rigid support. The time taken by the transverse wave formed in the rope to travel through the full length of the rope is (Assume g = 9.8 m/s²)

1) 0.5 s   2) 1.6 s   3) 1.2 s   4) 1.0 s

Key (4)

Sol :- Time taken $= 2\sqrt{\frac{L}{g}}$

$$= 2\sqrt{\frac{2.45}{9.8}} = 1 \sec$$

92. When a vibrating tuning fork is placed on a sound box of a sonometer, 8 beats per second are heard when the length of the sonometer wire is kept at 101 cm or 100 cm. Then the frequency of the tuning fork is (consider that the tension in the wire is kept constant)

1) 1616 Hz   2) 1608 Hz   3) 1632 Hz   4) 1600 Hz

Key (2)

Sol :- $(101)(n-8) = (100)(n+8)$

$n = 1608$ Hz

93. The objective and eyepiece of an astronomical telescope are double convex lenses with refractive index 1.5, When the telescope is adjusted to infinity, the separation between the two lenses is 16cm. If the space between the lenses is now filled with water and again telescope is adjusted for infinity, then the present separation between the lenses is

1) 8 cm   2) 16 cm   3) 24 cm   4) 32 cm

Key (4)

Sol :-   air   air $f = f_0$

When both sides of lens is air, focal lengths are f_0 and f_e. When one side is air and other sides is water, focal lengths are $2f_0$ and $2f_e$

$\therefore$ seperation (final) $= 2(f_0 + f_e)$

$= 32$ cm

94. The dispersive powers of the materials of two lenses forming an achromatic combination are in the ratio of 4 : 3. Effective focal length of the two lenses is +60 cm then the focal lengths of the lenses should be

1) -20 cm, 25 cm   2) 20 cm, -25 cm   3) -15 cm, 20 cm   4) 15 cm, -20 cm

Key (4)

Sol :- $\dfrac{W_1}{f_1} + \dfrac{W_2}{f_2} = 0$

$\dfrac{4}{f_1} + \dfrac{3}{f_2} = 0$

and $\dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1}{60}$

solving $f_1 = 15cm$ $f_2 = -20cm$

95. Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between them is 2λ as shown in figure. The first bright fringe is formed at 'P' due to interference on a screen placed at a distance 'D' from S_1 ($D >> \lambda$), then OP is
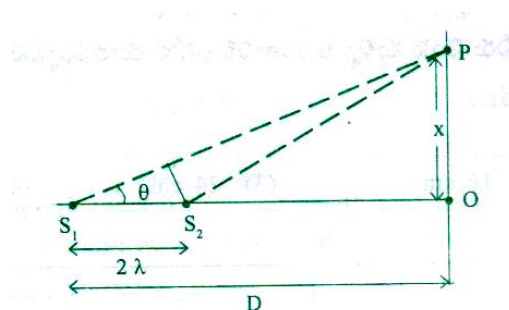
1) $\sqrt{2}\,D$ 2) 1.5 D   3) $\sqrt{3}\,D$ 4) 2D

Key (3)

Sol :- Path difference = $2\lambda\cos\theta = \Delta x$

$2\lambda\cos\theta = n\lambda$ ($n=1$ *for first bright fringe*)

$\therefore \cos\theta = \dfrac{1}{2} = \theta = 60^0$

$\tan\theta = \tan 60^0 = \dfrac{OP}{D}$

$\therefore OP = \sqrt{3}D$

96. A short bar magnet in a vibrating magnetometer makes 16 oscillations in 4 seconds. Another short magnet with same length and width having moment of inertia 1.5 times the first one is placed over the first magnet and oscillated. Neglecting the induced magnetization, the time period of the combination is

    1) $2\sqrt{10}\,s$ 2) $20\sqrt{10}\,s$ 3) $\dfrac{2}{\sqrt{10}}s$ 4) $\dfrac{2.5}{\sqrt{10}}s$

No Answer

Sol :- $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$\dfrac{4}{16} = 2\pi\sqrt{\dfrac{I}{MB}}$

$T_2 = 2\pi\sqrt{\dfrac{I+1.5I}{MB}}$

$\therefore T_2 = \dfrac{1.25}{\sqrt{10}}\sec$

97. A magnetic needle lying parallel to a magnetic field is turned through 60^0. The work done on it is w. The torque required to maintain the magnetic needle in the position mentioned above is

    1) $\sqrt{3}\,w$ 2) $\dfrac{\sqrt{3}}{2}w$ 3) w/2   4) 2 w

Key (1)

Sol :- $W = MB(1-\cos\theta)$

$W = MB(1-\cos 60^0) = \dfrac{MB}{2}$

$MB = 2W$

$T = MB\sin 60^0 = \dfrac{\sqrt{3}MB}{2} = \sqrt{3}W$

98. A parallel plate capacitor has a capacity 80×10^{-6} F when air is present between the plates. The volume between the plates is then completely with a dielectric slab of dielectric constant 20. The capacitor is now connected to a battery of 30 V by wires. The dielectric slab is then removed. Then, the charge that passes now through the wire is

    1) 45.6×10^{-3} C   2) 258.3×10^{-3} C   3) 120×10^{-3} C   4) 12×10^{-3} C

Key (1)

Sol :- Change passing through wire

    = KCV - CV

    = (K-1)CV

    = $(20-1) \times 80 \times 10^{-6} \times 30$

    = 45.6×10^{-3} C

99. Two small spheres each having equal positive charge Q (Coulomb) on each are suspended by two insulating strings of equal length L (meter) from a rigid hook (shown in Fig.). The whole set up is taken into satellite where there is no gravity. The two balls are now held by electrostatic forces in horizontal position, the tension in each string is then
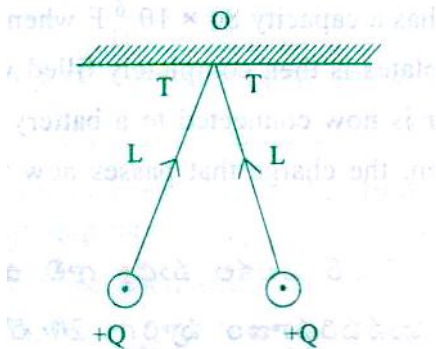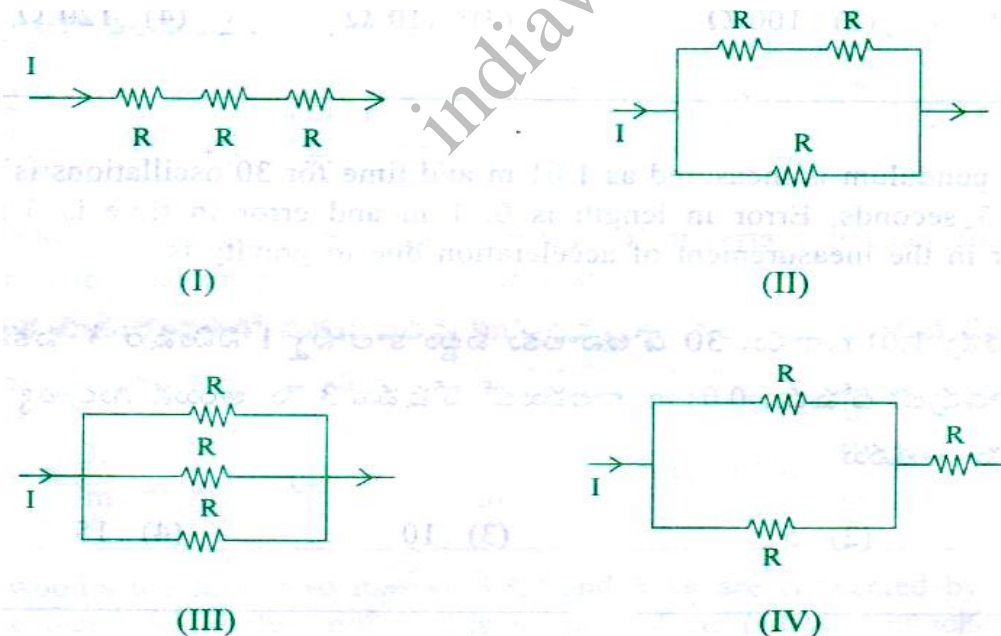

1) $\dfrac{Q^2}{16\pi\varepsilon_0 L^2}$ 2) $\dfrac{Q^2}{8\pi\varepsilon_0 L^2}$ 3) $\dfrac{Q^2}{4\pi\varepsilon_0 L^2}$ 4) $\dfrac{Q^2}{2\pi\varepsilon_0 L^2}$

Key (1)

Sol :- The angle between the threads is 180^0 in satellite

Therefore tension $T = \dfrac{Q^2}{16\pi\varepsilon_0 L^2}$

100. Three resistances of equal values are arranged in four different configurations as shown below. Power dissipation in the increasing order is

(I)   (II)

(III)   (IV)

1) (III) < (II) < (IV) < (I)   2) (II) < (III) < (IV) < (I)
3) (I) < (IV) < (III) < (II)   4) (I) < (III) < (II) < (IV)

Key (1)

Sol :- Effective resistances are

$$3R, \frac{2R}{3}, \frac{R}{3}, \frac{3R}{2}$$

Power = I^2 (Reffective)

101. Four resistors A, B, C and D form a Wheatstones bridge. The bridge us balanced when $C = 100\,\Omega$. If A and B are interchanged, the bridge balances for $C = 121\,\Omega$. The value of D is

1) $10\,\Omega$ 2) $100\,\Omega$ 3) $110\,\Omega$ 4) $120\,\Omega$

Key (3)

Sol :- $\dfrac{A}{B} = \dfrac{C}{D}$ $\qquad \therefore \dfrac{A}{B} = \dfrac{100}{D} - - - - - (1)$

$$\frac{B}{A} = \frac{121}{D} - - - - - (2)$$

$$\therefore D = 110\Omega$$

102. The length of a pendulum is measured as 1.01 m and time for 30 oscillations is measured as one minute 3 seconds. Error in length is 0.01 m and error in time is 3 secs. The percentage error in the measurement of acceleration due to gravity is

1) 1 2) 5 3) 10 4) 15

Key (3)

Sol :- $g = 4\pi^2 \dfrac{L}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$$\frac{\Delta g}{g} = \frac{0.01}{1.01} + 2\left(\frac{3}{63} \times \frac{1}{30}\right)$$

$$\frac{\Delta g}{g}\% \cong 10\%$$

103. Sum of magnitudes of two forces acting at a point is 16 N. If their resultant is normal to smaller force, and has a magnitude 8 N, then the forces are

1) 6 N, 10 N 2) 8 N, 8 N 3) 4 N, 12 N 4) 2 N, 14N

Key (1)

Sol :- $P + Q = 16N$

$$Q^2 - P^2 = 64$$

$$Q - P = 4$$

$$\therefore P = 6N \qquad Q = 10N$$

104. It is possible to project a particle with a given velocity in two possible ways so as to make them pass through a point P at a horizontal distance r from the point of projection, If t_1 and t_2 are times taken to reach this point in two possible ways, then the product $t_1 t_2$ is proportional to

1) $\dfrac{1}{r}$ 2) r 3) r^2 4) $\dfrac{1}{r^2}$

Key (2)

Sol :- $t_1 = \dfrac{2u\sin\theta}{g}$

$$t_2 = \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2} \propto r$$

105. The velocity 'v' reached by a car of mass 'm' at certain distance from the starting point driven with constant power 'P' is such that

1) $v \alpha \dfrac{3P}{m}$ 2) $v^2 \alpha \dfrac{3P}{m}$ 3) $v^3 \alpha \dfrac{3P}{m}$ 4) $v \alpha \left(\dfrac{3P}{m}\right)^2$

Key (2)

Sol :- Power = constant

$$P = mv\left(\dfrac{dv}{dt}\right)$$

$$\therefore mv\,dv = p\ dt$$

$$\therefore V^2 \propto \dfrac{3P}{m}$$

106. In Atwood's machine, two masses 3 kg and 5 kg are connected by a light string which passes over a frictionless pulley. The support of the pulley is attached to the ceiling of a compartment of a train. If the train moves in a horizontal direction with a constant acceleration 8 ms^{-2}, the tension in the string in Newtons is (g = 10 ms^{-2})

1) 3.75 2) 7.5 3) 15 4) 20

No Answer

Sol :- $T = \dfrac{2m_1 m_2}{m_1 + m_2}\left(\sqrt{a^2 + g^2}\right)$

$$\therefore T \cong 48N$$

107. A ball 'A' of mass 'm' moving along positive x-direction with kinetic energy 'K' and momentum P undergoes elastic head on collision with a stationary ball B of mass 'M'. After collision the ball A moves along negative X-direction with kinetic energy $\dfrac{K}{9}$. Final momentum of B is

1) P 2) $\dfrac{P}{3}$ 3) $\dfrac{4P}{3}$ 4) 4P

Key : 3

Sol :- <u>Before Collision:</u> $P_A = P$ $K_A = K$

$$P_B = zero$$

<u>After collision :</u> $K_A^{'} = K/9$ $P_A^{'} = -P/3$

From momentum consevation

$$P_B^{'} = 4P/3$$

108. Choose the correct statement

A) The position of centre of mass of a system is dependent on the choice of co-ordinate system

B) Newton's second law of motion is applicable to the centre of mass of the system

C) When no external force acts on a body, the acceleration of centre of mass is zero.

D) Internal forces can change the state of centre of mass

1) Both (A) and (B) are correct 2) Both (B) and (C) are wrong

3) Both (A) and (C) are wrong 4) Both (A) and (D) are wrong

Key : 4

Sol :- Option (B) and (C) are right

109. When the engine is switched off a vehicle of mass 'M' is moving on a rough horizontal road with momentum P. If the coefficient of friction between the road and tyres of the vehicle is μ_k, the distance travelled by the vehicle before it comes to rest is

1) $\dfrac{P^2}{2\mu_k M^2 g}$ 2) $\dfrac{2\mu_k M^2 g}{P^2}$ 3) $\dfrac{P^2}{2\mu_k g}$ 4) $\dfrac{P^2 M^2}{2\mu_k g}$

Key: 1

Sol :- $S = \dfrac{u^2}{2ukg}$ $P = mu$

$\therefore S = \dfrac{P^2}{m^2(2ukg)}$

110. **Assertion (A) :** The moment of inertia of a steel sphere is larger than the moment of inertia of a wooden sphere of same radius.
   Reason (R) : Moment of inertia is independent of mass of the body.
   The correct one is
   1) Both (A) and (R) are true, and (R) is the correct explanation of (A)
   2) Both (A) and (R) are true, but (R) is not the correct explanation of (A)
   3) (A) is true but (R) is wrong
   4) (A) is wrong but (R) is true

Key :3

111. Two solid spheres A and B each of radius 'R' are made of materials of densities ρ_A and ρ_B respectively.

Their moments of inertia about a diameter are I_A and I_B respectively. The value of $\dfrac{I_A}{I_B}$ is

1) $\sqrt{\dfrac{\rho_A}{\rho_B}}$ 2) $\sqrt{\dfrac{\rho_B}{\rho_A}}$ 3) $\dfrac{\rho_A}{\rho_B}$ 4) $\dfrac{\rho_B}{\rho_A}$

Key : 3

Sol :- $\dfrac{I_A}{I_B} = \dfrac{M_A}{M_B} = \dfrac{\rho_A}{\rho_B}$ $(\because R = same)$

112. Match column A (layers in the ionosphere for skywave propagation) with column B (their height range):

Column A	Column B
(I) D- layer	(a) 250-400 km
(II) E-layer	(b) 170-190 km
(III) F_1 -layer	(c) 95-120 km
(IV) F_2 - layer	(d) 65-75 km

The correct answer is

	(I)	(II)	(III)	(IV)
1)	(a)	(b)	(c)	(d)
2)	(d)	(c)	(a)	(b)
3)	(d)	(c)	(b)	(a)
4)	(c)	(d)	(c)	(b)

Key : 3

113. In a transistor if $\dfrac{I_C}{I_E} = \alpha$ and $\dfrac{I_C}{I_B} = \beta$. If α varies between $\dfrac{20}{21}$ and $\dfrac{100}{101}$, then the value of β lied between

1) 1-10   2) 0.95-0.99   3) 20-100   4) 200-300

Key : 3

Sol: - $\beta = \dfrac{\alpha}{1-\alpha}$

114. The half life of a radioactive element is 10 hours. The fraction of initial radioactivity of the element that will remain after 40 hours is

1) $\dfrac{1}{2}$ 2) $\dfrac{1}{16}$ 3) $\dfrac{1}{8}$ 4) $\dfrac{1}{4}$

Key : 2

Sol : - $\dfrac{A}{A_0} = \dfrac{1}{2^n}$ $n = 4$

115. The half life of Ra^{226} is 1620 years. Then the number of atoms decay in one second in 1 gm of radium (Avogadro number = 6.023×10^{23})

1) 4.23×10^9 2) 3.16×10^{10} 3) 3.61×10^{10} 4) 2.16×10^{10}

Key : 3

Sol :- $\dfrac{dN}{dt} = \lambda N$

$$= \left(\dfrac{0.693}{1620 \times 365 \times 86400} \right) \times \left(\dfrac{1}{226} \times 6.023 \times 10^{23} \right) = 3.61 \times 10^{10}$$

116. A proton when accelerated through a potential difference of V, has a de Broglie wavelength λ associated with it. If an alpha particle is to have the same de Broglie wavelenth λ, it must be accelerated through a potential difference of

1) $\dfrac{V}{8}$ 2) $\dfrac{V}{4}$ 3) 4V   4) 8V

Key : 1

Sol :- For proton : $\lambda = \dfrac{h}{\sqrt{2meV}}$

For α particle: $\lambda = \dfrac{h}{\sqrt{2 \times 4m \times 2eV}}$

$\therefore\ V' = V/8$

117. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^8 m/s is equal to that of a photon. The ratio of kinetic energy of the electron to that of the photon $\left(C = 3 \times 10^8\ m/s \right)$

1) 2   2) 4   3) $\dfrac{1}{2}$ 4) $\dfrac{1}{4}$

Key : 4

Sol :- $ratio = \dfrac{K}{(hc/\lambda)}$ $---(1)$ $\lambda = \dfrac{h}{\sqrt{2mk}}$ (for electron)

$= \dfrac{V}{2C}$ $\qquad K = \dfrac{1}{2}mV^2$ (for electron)

118. A primary coil and secondary coil are placed close to each other. A current, which changes at the rate of 25 amp in a millisecond, is present in the primary coil. If the mutual inductance is 92×10^{-6} Henries, then the value of induced emf in the secondary coil is

   1) 4.6 V   2) 2.3 V   3) 0.368 mV   4) 0.23 mV

Key : 2

Sol :- Induced emf $e = -M\dfrac{di}{dt}$

$ = 92 \times 10^{-6} \times \dfrac{25}{10^{-3}} \; Volt$

$e = 2.3 \; Volt$

119. A long curved conductor carries a current $\overline{I}$ ($\overline{I}$ is a vector). A small current element of length $\overline{d\ell}$, on the wire induces a magnetic field at a point, away from the current element. If the position vector between the current element and the point is $\overline{r}$, making an angle with current element then, the induced magnetic field density; $\overline{dB}$ (vector) at the point is (μ_0 = permeability of free space)

   1) $\dfrac{\mu_0 I \overline{d\ell} \times \overline{r}}{4\pi r}$ perpendicular to the current element $\overline{d\ell}$

   2) $\dfrac{\mu_0 \overline{I} \times \overline{r} \times \overline{d\ell}}{4\pi r^2}$ perpendicular to the current element $\overline{d\ell}$

   3) $\dfrac{\mu_0 \overline{I} \times \overline{d\ell}}{r}$ perpendicular to the plane containing the current element and position vector $\overline{r}$

   4) $\dfrac{\mu_0 \overline{I} \times \overline{d\ell}}{4\pi r^2}$ perpendicular to the plane containing current element and position vector $\overline{r}$

Question not correct

120. Total emf produced in a thermocouple does not depend on
   1) the metals in the thermocouple
   2) thomson coefficients of the metals in the thermocouple
   3) temperature of the junctions
   4) the duration of time for which the current is passed through thermocouple

Key: 4

CHEMISTRY

121. Which one of the following elements, when present as an impurity in silicon makes it a p-type semiconductor ?

   (1) As   (2) P   (3) In   (4) Sb

Key : 3

Sol : P-type semi conductors need trivalent impurity

122. Which one of the following statements is correct for the reaction

$$CH_3COOC_2H_5 + NaOH \rightarrow CH_3COONa + C_2H_5OH$$
$$\underset{(aq)}{} \qquad \underset{(aq)}{} \qquad \underset{(aq)}{} \qquad \underset{(aq)}{}$$
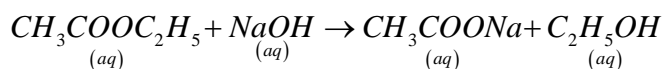
    (1) Order is two but molecularity is one   (2) Order is one but molecularity is two

    (3) Order is one and molecularity is one   (4) Order is two and molecularity is two

Key : 4

Sol : $rate = k[ester][NaOH]$

123. The catalyst and promoter respectively used in the Haber's process of industrial synthesis of ammonia are

    (1) Mo, V_2O_5 (2) V_2O_5, Fe (3) Fe, Mo (4) Mo, Fe

Key : 3

Sol : Catalyst - Iron; promoter molybdinum

124. Which one of the following statements is **NOT** correct ?

    (1) The pH of $1.0 \times 10^{-8} M \ HCl$ is less than 7

    (2) The ionic product of water at $25^0 C$ is $1.0 \times 10^{-14} \ mol^2 L^{-2}$

    (3) Cl^- is a Lewis acid

    (4) Bronsted - Lowry theory cannot explain the acidic character of $AlCl_3$

Key : 3

Sol : Cl$^-$ is a lewis base

125. The molar heat capacity $\left(C_p\right)$ of water at constant pressure is $75 \ J.K^{-1}.mol^{-1}$. The increase in temperature

    $\left(in \ K\right)$ of $100 g$ of water when 1 k.J. of heat is supplied to it is.

    (1) 2.4   (2) 0.24   (3) 1.3   (4) 0.13

Key : 1

Sol : Molar heat capacity = specific heat X Molecular weight

126. Gelly is a colloidal solution of

    (1) Solid in liquid   (2) Liquid in solid   (3) Liquid in liquid   (4) Solid in solid

Key : 2

Sol : Gel is a colloidal solution having liquid dispersed phase in solid dispersion medium

127. The product(s) formed when H_2O_2 reacts with disodium hydrogen phosphate is (are)

    (1) P_2O_5, Na_3PO_4 (2) $Na_2HPO_4.H_2O_2$ (3) $NaH_2PO_4.H_2O$ (4) $Na_2HPO_4.H_2O$

Key : 2

Sol : Hydrogen peroxide form addition compound with disodium hydrogen peroxide

128. Which of the following is **NOT** correct ?

    (1) LiOH is a weaker base than NaOH   (2) Salts of Be undergo hydrolysis

    (3) $Ca\left(HCO_3\right)_2$ is soluble in water   (4) Hydrolysis of beryllium carbide gives acetylene

Key : 4

Sol : Beryllium carbide on hydrolysis produce methane

129. What is Z in the following reactions ?

$$BCl_3 + H_2 \xrightarrow[450^0C]{Cu-Al} X + HCl$$

$$X \xrightarrow{methylation} Z$$
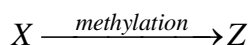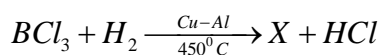
(1) $(CH_3)BH_2$ (2) $(CH_3)_4 B_2H_2$ (3) $(CH_3)_3 B_2H_3$ (4) $(CH_3)_6 B_2$

Key : 2

Sol : Diborane has two bridged hydrogens

130. Which one of the following elements reduces $NaOH$ to Na ?

(1) Si (2) Pb (3) C (4) Sn

Key : 3

Sol : $6NaOH + 2C \longrightarrow 2Na_2CO_3 + 2Na + 3H_2$
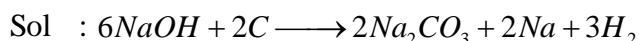
131. Which one of the following is used in the preparation of cellulose nitrate ?

(1) KNO_3 (2) HNO_3 (3) KNO_2 (4) HNO_2

Key : 2

Sol : HNO_3 acts as nitrating agent

132. The oxoacid of sulphur which contains two sulphur atoms in different oxidation states is.

(1) Pyrosulphurous acid   (2) Hyposulphurous acid   (3) Pyrosulphuric acid   (4) Persulphuric acid

Key : 1

Sol : $H_2S_2O_5$, $HO-\overset{\overset{O}{\parallel}}{\underset{\underset{O}{\parallel}}{S}}-\overset{\overset{O}{\parallel}}{S}-OH$ has two sulphur atoms are in different envoronment
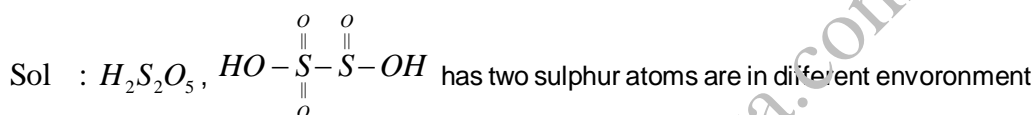
133. Bond energy of Cl_2, Br_2 and I_2 follow the order.

(1) $Cl_2 > Br_2 > I_2$ (2) $Br_2 > Cl_2 > I_2$ (3) $I_2 > Br_2 > Cl_2$ (4) $I_2 > Cl_2 > Br_2$

Key : 1

Sol : $Cl_2 > Br_2 > I_2$
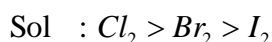
134. **Assertion (A) :** The boiling points of noble gases increases from He to Xe.

Reason (R) : The interatomic vander Waals attractive forces increase from He to Xe.

The correct answer is

(1) Both (A) and (R) are true, and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true, and (R) is not the correct explanation of (A)

(3) (A) is true but (R) is not true

(4) (A) is not true but (R) is ture

Key : 1

Sol : vanderwaals forces directly proportional to molar mass and surface area

135. A coordinate complex contains Co^{3+}, Cl^- and NH_3. When dissolved in water, one mole of this complex gave a total of 3 moles of ions. The complex is.

(1) $\left[Co(NH_3)_6\right]Cl_3$ (2) $\left[Co(NH_3)_5 Cl\right]Cl_2$

(3) $\left[Co(NH_3)_4 Cl_2\right]Cl$ (4) $\left[Co(NH_3)_3 Cl_3\right]$

Key : 2

Sol : $[Co(NH_3)_5 Cl]Cl_2$ produce 3 ions in water
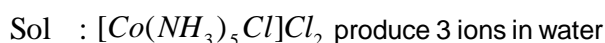
136. Ni anode is used in the electrolytic extraction of

(1) Al (2) Mg

(3) Na by Down's process   (4) Na by Castner's process

Key : 4

Sol : In castner process Ni acts as anode

137. The pair of gases responsible for acid rain are.

(1) H_2, O_3 (2) H_4C, O_3 (3) NO_2, SO_2 (4) CO, CH_4

Key : 3

Sol : Nitrogen and sulphur oxides cause acid rain

138. The chlorination of ethane is an example for which type of the following reactions ?

(1) Nucleophilic substitution   (2) Electrophilic substitution

(3) Free redical substitution   (4) Rearrangement

Key : 3

Sol : Free radical mechanism

139. Different conformations of the same molecule are called

(1) Isomers   (2) Epimers   (3) Enantiomers   (4) Rotamers

Key : 4

Sol : Conformes or conformational isomers are also known as rotamers

140. Which of the following statement is **NOT** correct ?

(1) The six carbons in benzene are sp^2 hybridised

(2) Benzene has $(4n + 2)\pi$ electrons

(3) Benzene undergoes substitution reactions

(4) Benzene has two carbon - carbon bond lengths, $1.54 \overset{0}{A}$ and $1.34 \overset{0}{A}$

Key : 4

Sol : Benzene has same c-c bond length due to resonance

141. Match the following

List-I	List-II
(A) Acetaldehyde, Vinylalcohol	(I) Enantiomers
(B) Eclipsed and staggered ethane	(II) Tautomers
(C) $(+)2 -$ Butanol, $(-)2 -$ Butanol	(III) Chain isomers
(D) Methyl -n- propylamine and Diethylamine	(IV) Conformational isomers
	(V) Metamers

The correct answer is.

	(A)	(B)	(C)	(D)
(1)	(II)	(IV)	(III)	(V)
(2)	(II)	(IV)	(I)	(V)
(3)	(V)	(I)	(IV)	(II)
(4)	(V)	(I)	(III)	(II)

Key : 2

Sol : (A) $CH_3 - CHO \rightleftharpoons H_2C = CH - OH$ ------- Tautomers

(B) Eclipsed and staggered ethane   -------- Conformational isomers

(C) (+) and (-) Butanols are   ------- Optical Isomers (Enantiomers)

(D) $CH_3 - NH - CH_2 - CH_2 - CH_3$ and $C_2H_5 - NH - C_2H_5$ ------ Metamers

142. With respect to chlorobenzene, which of the following statements is **NOT** correct ?

(1) Cl is ortho/para directing

(2) Cl exhibits $+M$ effect

(3) Cl is ring deactivating

(4) Cl is meta directing

Key : 4

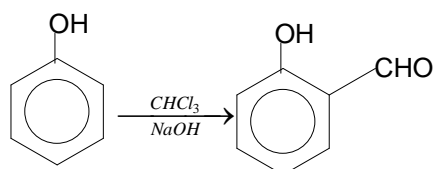Sol : Chlorine has non bonding pair of electrons

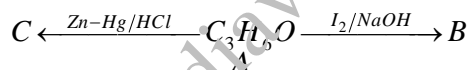143. Identify the product in the following reaction.
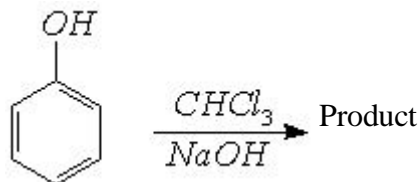
Key : 2

Sol : Reimer -Tiemann reaction

144. Compound -A $\left(C_3H_6O\right)$ undergoes following reactions to form B and C. Identify A, B and C

$$C \xleftarrow{Zn-Hg/HCl} \underset{A}{C_3H_6O} \xrightarrow{I_2/NaOH} B$$

A	**B**	**C**
(1) $H_3C - \overset{O}{\overset{\|}{C}} - CH_3$	CHI_3	$H_3C - CH_2 - CH_3$
(2) $\underset{H}{H_2C = \overset{\|}{C} - CH_2OH}$	CH_3I	$H_3C - CH_2 - CH_2 - OH$
(3) $H_3C - CH_2 - CHO$	CH_3I	$H_3C - \underset{OH}{\overset{\|}{CH}} - CH_3$
(4) $H_3C - \overset{O}{\overset{\|}{C}} - CH_3$	CHI_3	$H_3C - \underset{OH}{\overset{\|}{CH}} - CH_3$

Key : 1

Sol : $H_3C - CO - CH_3 \xrightarrow{Zn-Hg/HCl} \Pr opane(A)$

$H_3C - CO - CH_3 \xrightarrow{I_2/NaOH} CHI_3(B) + H_3CCOONa$
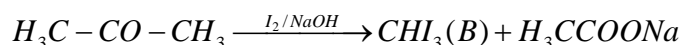
145. What is the product obtained in the reaction of Acetaldehyde with semicarbazide ?

(1) $H_3C - CH = N - NH - \overset{\overset{O}{\|}}{C} - NH_2$
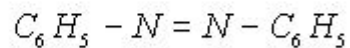
(2) $H_3C - CH = N - NH_2$

(3) $H_3C - CH = N - OH$

(4) $H_3C - \underset{\underset{CH_3}{|}}{C} = N - NH - \overset{\overset{O}{\|}}{C} - NH_2$

Key : 1

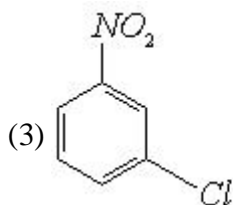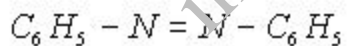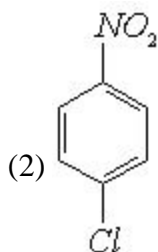Sol : $H_3C - CHO + H_2N - NH - CO - NH_2 \longrightarrow H_3C - CH = N - NH - CO - NH_2$
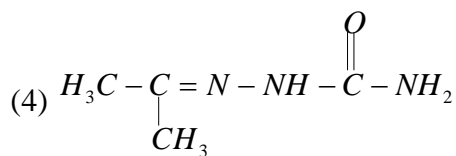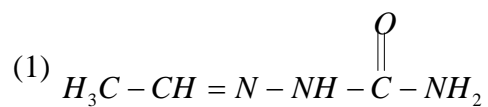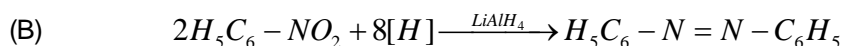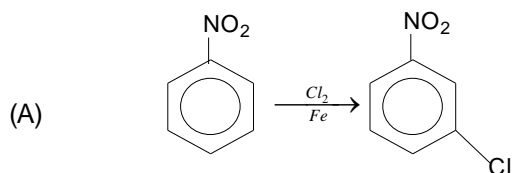
146. Identify A and B in the following reactions

A   **B**

(1) $C_6H_5 - \underset{H}{N} - \underset{H}{N} - C_6H_5$

(2) $C_6H_5 - N = N - C_6H_5$

(3) $C_6H_5 - N = N - C_6H_5$

(4) $C_6H_5 - NH_2$
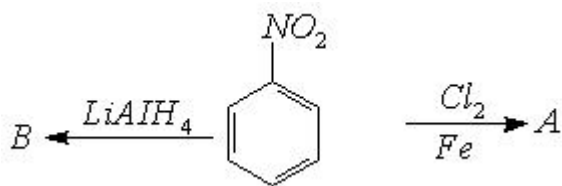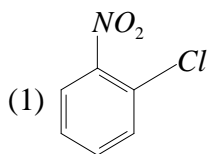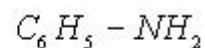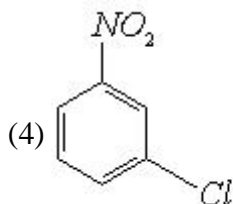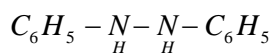
Key : 3

Sol : Nitro group is meta directing group

(A)

(B) $2H_5C_6 - NO_2 + 8[H] \xrightarrow{LiAlH_4} H_5C_6 - N = N - C_6H_5$

147. The monomer of neoprene is
(1) 1, 3-Butadiene
(2) 2-Chloro -1, 3-Butadiene
(3) 2-Methyl -1, 3-butadiene
(4) Vinyl chloride

Key : 2

Sol : Neoprene is a polymer of chloroprene; $H_2C = C - CH = CH_2$ with Cl on the second carbon

148. The site of action of insulin is
(1) Mitochondria　　(2) Nucleus　　　(3) Plasma membrane　　(4) DNA

Key : 3

Sol : Insulin is a peptide hormone

149. 4-Hydroxy acetanilide belongs to which of the following ?
(1) Antipyretic　　(2) Antacid　　　(3) Antiseptic　　　(4) Antihistamine

Key : 1

Sol : Paracetamol is antipyretic

150. In photoelectric effect, if the energy required to overcome the attractive forces on the electron, (work functions) of Li, Na and Rb are $2.41 eV, 2.30 eV$ and $2.09 eV$ respectively, the work function of 'K' could approximately be in eV
(1) 2.52　　　(2) 2.20　　　(3) 2.35　　　(4) 2.01

Key : 2

Sol : Work function values decrease down the group of elements in periodic table

151. The quantum number which explains the line spectra observed as doublets in case of hydrogen and alkali metals and doublets and triplets in case of alkaline earth metals is
1) Spin　　　2) Azimuthal　　　3) Magnetic　　　4) Principal

Key : 1

Sol : Uhlenbeck and Samuel Goudsmith proposed spin quantum number to explain atomic spectra

152. Which one of the following cannot form an amphoteric oxide?
1) Al　　　2) Sn　　　3) Sb　　　4) P

Key : 4

Sol : Oxides of phosphorous are acidic in nature

153. The formal charges of C and O atoms in $CO_2 \left(: \ddot{O} = C = \ddot{O} : \right)$ are, respectively
1) 1, -1　　　2) -1, 1　　　3) 2, -2　　　4) 0, 0

Key : 4

Sol : $: \ddot{O} = C = \ddot{O} :$
formalcharge = group no - [no. of bonds + non bonding electrons]

154. According to molecular orbital theory, the total number of bonding electron pairs in O_2 is
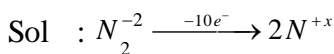1) 2　　　2) 3　　　3) 5　　　4) 4

Key : 3

Sol : $\sigma 1s^2 < \sigma 1s^2* < \sigma 2s^2 < \sigma 2s^2* < \sigma 2p^2_x < \pi 2p^2_y = \pi 2p^2_z < \pi 2p^1*_y = \pi 2p^1 z*$

155. One mole of N_2H_4 loses 10 moles of electrons to form a new compound Z. Assuming that all the nitrogens appear in the new compound, what is the oxidation state of nitrogen in Z? (There is no change in the oxidation state of hydrogen)

    1) -1   2) -3   3) +3   4) +5

Key : 3

Sol : $N_2^{-2} \xrightarrow{-10e^-} 2N^{+x}$

156. Which one of the following equations represents the variation of viscosity of coefficient (η) with temperature (T)?

    1) $\eta = Ae^{-E/RT}$ 2) $\eta = Ae^{E/RT}$ 3) $\eta = Ae^{-E/KT}$ 4) $\eta = Ae^{-E/T}$

Key : 2

Sol : $\eta = A.e^{\frac{E}{RT}}$

157. The weight in grams of a non-volatile solute (M. wt : 60) to be dissolved in 90 g of water to produce a relative lowering of vapour pressure of 0.02 is

    1) 4   2) 8   3) 6   4) 10

Key : 3

Sol : $\dfrac{p0 - ps}{p0} = \dfrac{w_{solute}}{M_{solute}} X \dfrac{M_{solvent}}{W_{solvent}}$

158. The experimentally determined molar mass of a non-volatile solute, $BaCl_2$ in water by Cottrell's method, is

    1) equal to the calculated molar mass   2) more than the calculated molar mass

    3) less than the calculated molar mass   4) double of the calculated molar mass

Key : 3

Sol : $BaCl_2$ undergoes ionisation and produce 3 particles

159. The number of molar of electrons required to deposit 36 g of Al from an aqueous solution of $Al(NO_3)_3$ is (At. wt. of Al=27)

    1) 4   2) 2   3) 3   4) 1

Key : 1

Sol : $\dfrac{Weight}{Equivalent weight} = \dfrac{Q}{F}$

160. The emf (in V) of a Daniel cell containing 0.1 M $ZnSO_4$ and 0.01 M $CuSO_4$ solutions at their respective electrodes is

    $\left(E^o_{Cu^{2+}/Cu} = 0.34V; E^o_{Zn^+/Zn} = -0.76V \right)$

    1) 1.10   2) 1.16   3) 1.13   4) 1.07

Key : 4

Sol : $E_{electrode} = E^0_{electrode} + \dfrac{0.06}{2} \log[M^{n+}]$

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